Review 2 Fall 1989

1. Give the form of the partial fraction decomposition for \( f(x) \) without solving for the constants:

\[
f(x) = \frac{x^4 + 2x^3 + 6x}{(x - 2)^2(x + 2)(x^2 + 4)^2}
\]

2. Evaluate:

a) \( \int \frac{2x^2 - x + 2}{x^2(x^2 + 1)} \, dx \)

b) \( \int \frac{x^3 + 2}{x^2 - 1} \, dx \)

c) \( \int \frac{dy}{y^2\sqrt{y^2 - 7}} \)

d) \( \int \frac{dx}{\sqrt{16 + 9x^2}} \)

e) \( \int \frac{x - 1}{\sqrt{2x - x^2}} \, dx \)

3. Use Simpson’s rule with \( n = 4 \) to write a sum which approximates the integral

\[
\int_2^6 \sqrt{x^2 - 1} \, dx
\]

4. 

a) Convert to Cartesian coordinates and identify the curve \( r = 4 \cos \theta - 2 \sin \theta. \)

b) Find the equation in polar coordinates for the curve whose cartesian equation is

\( 4x^2 + 9y^2 = 1. \)

5. Graph the curve whose equation is \( r = 4 \sin 3\theta \)

6. Sketch the following curves by eliminating the parameter \( t \), and label the direction of increasing \( t \):

a) \( x = 2 \cos t, \quad y = 5 \sin t, \quad 0 \leq t \leq 2\pi \)

b) \( x = \sqrt{t}, \quad y = 3t + 5 \)

7. Let \( C \) be the curve \( x = \frac{t^2}{2}, \quad y = \frac{1}{3}, \quad t \geq 0. \)

a) Show that the points \( (0, \frac{1}{3}) \) and \( (8,9) \) lie on \( C \) and find the corresponding values for \( t. \)

b) Find the arc length along \( C \) from \( (0, \frac{1}{3}) \) to \( (8,9). \)

c) Find an equation for the tangent line to \( C \) at the point where \( t = 4. \)

d) Find \( \frac{dy}{dx} \) for \( C \) in terms of \( t. \)

8. Determine convergence or divergence, and evaluate the limits which exist. Give reasons, and indicate each time you apply L’Hôpital’s rule:

a) \( \lim_{x \to \infty} (1 + \frac{6}{x})^x \)

b) \( \lim_{x \to 0^+} x \ln(\sin x) \)

c) \( \lim_{x \to \infty} (1 + x)^{\frac{1}{x}} \)

d) \( \int_0^8 \frac{dx}{(x - 4)^2} \)