Final Review Spring 1990

1. Simplify
   a) $e^{3\ln x + 2 \ln 5}$    b) $\tan^{-1}\left(\tan\frac{4\pi}{3}\right)$    c) $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

2. Find $x$ if $\ln\left(\frac{1}{x^2}\right) = e$

3. Find $\frac{dy}{dx}$ if
   a) $y = x(\sin^{-1} x)^2$    b) $y = x(x-e^x)$
   c) $y = \frac{4x^2 + 5}{(x^2 + 1)^3\sqrt{x-3}}$ (use logarithmic differentiation)

4. Solve the differential equation $\frac{dy}{dx} = 2x(y^2 + 1)$ with $y(0) = -1$

5. Find $\frac{dy}{dx}\bigg|_{t=1}$ and $\frac{d^2y}{dx^2}\bigg|_{t=1}$ for
   \[
   \begin{cases}
   x = t^7 + t^3 - 2 \\
   y = 4t^2 + 5
   \end{cases}
   \]

6. Integrate
   a) $\int_1^e \frac{\ln x}{\sqrt{x}} \, dx$    b) $\int_{-1}^2 \frac{1}{x} \, dx$    c) $\int \sin^4 x \cos^5 x \, dx$
   d) $\int \frac{x+1}{\sqrt{x^2 + 2x + 5}} \, dx$    e) $\int \frac{2x^2 - 3x - 1}{x^2 - 2x - 3} \, dx$    f) $\int_3^7 \frac{dx}{x^2 - 6x + 25}$

7. a) Sketch the curves $r = 1 + \sin \theta$ and $r = 3 \sin \theta$ on the same axes.
   b) Convert to rectangular coordinates $r^2 = \sin 2\theta$.

8. a) Use Simpson’s Rule with $n = 4$ to approximate the integral $\int_0^{\frac{1}{2}} \frac{dx}{1 + x^2}$.
   Express your answer as a sum. Do not simplify.
   b) Use Maclaurin Series to approximate the same integral so that the error in your approximation is less than $\frac{1}{100}$. Do not use a calculator.

9. Find
   a) $\lim_{x \to 0} \frac{\sin x + x}{x^2}$    b) $\lim_{x \to \infty} x\left(\frac{x^2 - 1}{x^4 + 1}\right)$
10. Determine convergence or divergence
   a) \[ \sum_{k=1}^{\infty} \sqrt{\frac{k-1}{k^3 + 6}} \]
   b) \[ \sum_{k=1}^{\infty} \frac{8^k}{2 \cdot 4 \cdot 6 \cdots (2k)} \]
   c) \[ \sum_{k=1}^{\infty} \frac{(-1)^k}{(1 + \frac{1}{k})^k} \]

11. Find the interval of convergence of \[ \sum_{k=1}^{\infty} \frac{(-1)^k (2x + 1)^k}{k + 1} \]

12.
   a) Find the Maclaurin Series for \( f(x) = \frac{1}{(1 + x)^2} \) by finding the derivatives of \( f^{(k)}(x) \).
   b) Find the Taylor expansion for the same \( f(x) \) about \( a = 1 \). Express both (a) and (b) in \( \sum \) notation.
   c) Find the Lagrange form of the remainder, \( R_{15}(\pi) \), for the same \( f(x) \) about \( a = 1 \).

13.
   a) Find the 3rd. Taylor polynomial for \( f(x) = \tan^{-1} x \) about \( a = 1 \).
   b) Find the Lagrange form of the remainder, \( R_2(\frac{\pi}{2}) \) for \( f(x) = \cos \pi x \) about \( a = 1 \).

14.
   a) How many terms of the alternating harmonic series are needed to approximate \( \ln 2 \)
      to two decimal place accuracy? Do this without a calculator.
   b) How many terms of the Maclaurin series for \( \sin x \) must be used to compute \( \sin(1) \)
      to within an accuracy of .0002? Do this without a calculator.

15. By integrating an appropriate series, find the Maclaurin series for \( \ln(1 + 4x) \).
    Express your answer in \( \sum \) notation.

16. Express in the form \( x = iy \), \( x, y \) real:
   a) \( \frac{3 + i}{1 - 2i} \)
   b) \( 3 \text{cis} \frac{\pi}{6} \)
   c) \( e^{2 + \frac{\pi i}{4}} \)
   d) \( (1 + i)^{10} \)

17. Find all complex sixth roots of \( -64 \). Leave your answer in the form \( r \text{cis} \theta \).
    \( [r \text{cis} \theta = r(\cos \theta + i \sin \theta)] \)

18. Find all values of \( z \) such that \( z^4 = -1 + \sqrt{3}i \). Plot and label the answers on a graph.

19. Compute the radius and center of the circle of convergence of
    \[ \sum_{k=1}^{\infty} \frac{(z + 3 - 4i)^k}{k^5 \cdot 5^k} \]
    Sketch the circle in the complex plane.