1. (4 points) Determine whether the spring modeled by 
\[(mD^2 + bD + k)x = 0\] 
with \(m = 1\) gram, \(k = 5\) dynes/cm and \(b = 4\) gram/s is undamped, underdamped, critically damped, or overdamped.

2. (6 points) Determine whether the system 
\[\begin{align*}
x' &= -ty - z + t^2 \\
y' &= -x - \frac{z}{t} + 1 \\
z' &= x - ty \\
w' &= tx - y\sqrt{3} + z\sin t + w
\end{align*}\] 
is linear. If it is linear
a. determine whether it is homogeneous,  
b. determine its order, and  
c. write it in matrix form.

3. (8 points) Given the differential equation 
\[(D^2 + 4D + 3)x = 3t + 7 \quad (N)\] 

a. find the equivalent system \((S_{N})\),  
b. the general solution of \((N)\) is 
\[x(t) = c_1e^{-t} + c_2e^{-3t} + t + 1. \text{ You do not need to verify this.}\] Use the general solution of \((N)\) to obtain each component of the general solution of \((S_{N})\),  
c. write \((S_{N})\) in matrix form,  
d. write the general solution of \((S_{N})\) in the form 
\[\vec{x} = c_1\vec{h}_1(t) + \cdots + c_n\vec{h}_n(t) + \vec{p}(t)\].

4. (8 points) In parts a. and b. you are given a matrix \(A\), a vector-valued function \(\vec{E}(t)\) and formulas describing a collection of solutions of the nonhomogeneous system \(D\vec{x} = A\vec{x} + \vec{E}(t)\). In each case decide whether the collection is complete.

a. \(A = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}\), \(\vec{E}(t) = \begin{pmatrix} 2e^{-t} \\ -e^{-t} \end{pmatrix}\) : 
\[\begin{align*}
x_1 &= 2c_1e^{-2t} + c_2e^{-t} \\
x_2 &= -c_1e^{-2t} - c_2e^{-t} + e^{-t}
\end{align*}\] 
\[x_1 = 6c_1e^{4t} - 2c_2e^{-4t}\] 
\[x_2 = 2c_1e^{4t} - 6c_2e^{-4t}\] 
\[x_3 = c_1e^{4t} + c_2e^{-4t} - 2\]  
b. \(A = \begin{pmatrix} 5 & -3 & 0 \\ 3 & -5 & 0 \\ 0 & 1 & 2 \end{pmatrix}\), \(\vec{E}(t) = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}\) : 
\[\begin{align*}
x_1 &= 4t + 1, \quad t < 12 \\
x_2 &= 0, \quad 12 \leq t < 30 \\
x_3 &= t^2, \quad t \geq 30
\end{align*}\]  

5. (6 points) Rewrite 
\[f(t) = \begin{cases} 4t + 1 & t < 12 \\ 0 & 12 \leq t < 30 \\ t^2 & t \geq 30 \end{cases}\]  
in unit step function notation.

Examination continues on next page
6. a. (5 points) Compute $\mathcal{L}[e^{1-2t}]$ using the definition. No credit by any other method.
   b. (3 points) State for which values of $s$ this Laplace transform is defined.

7. (8 points) Find $\cos 5t \ast 4$.

8. (8 points) Find the Laplace transform of $f(t) = 5te^{7t} \sin 2t$.

9. (16 points) Find the inverse Laplace transform of
   a. $\frac{s + 2}{s^2 + 4s + 5}$.
   b. $\frac{5s}{(s^2 + 25)^2}$.

10. (20 points) Solve using the Laplace transform. No credit by any other method.
    a. $x'' + 4x' + 4x = t^2e^{-2t}$, $x(0) = x'(0) = 0$.
    b. $(D - 1)x = \begin{cases} 0 & t < 2 \\ 1 & t \geq 2 \end{cases}$, $x(0) = 1$.

11. (8 points) Check this set of vectors for linear independence:
    $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 1 \end{pmatrix}$
    $\begin{pmatrix} 2 \\ 1 \\ 4 \\ 3 \\ 2 \end{pmatrix}$
    $\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$.

END OF EXAMINATION