1. (5 points) Determine whether the system

\[ \begin{align*}
    x' &= -ty - z + t \\
y' &= -\frac{x}{t} - \frac{z}{t} + 1 \\
z' &= x - ty
\end{align*} \]

is linear. If it is linear
a. determine whether it is homogeneous,  
   b. determine its order, and  
   c. write it in matrix form.

2. (5 points) Given the o.d.e. 

\[ (D^2 - 1)x = t \quad \text{ (N)} \]

a. find the equivalent system \((S_N)\),  
b. the general solution of \((N)\) is \(x(t) = c_1 e^{-t} + c_2 e^t - t\). \textbf{You do not need to verify this.}  
   Use the general solution of \((N)\) to obtain each component of the general solution of \((S_N)\),  
c. write \((S_N)\) in matrix form,  
d. write the general solution of \((S_N)\) in the form \(\bar{x} = c_1 \bar{h}_1(t) + \cdots + c_n \bar{h}_n(t) + \bar{p}(t)\).

3. (5 points) Determine whether the spring modeled by \((mD^2 + bD + k)x = 0\) with \(m = 1\) gram, \(k = 4\) dynes/cm and \(b = 4\) gram/s is undamped, underdamped, critically damped, or overdamped.

4. (5 points) Rewrite \(f(t) = \begin{cases} 
                           4t + 1 & t < 2 \\
                           9 & 2 \leq t < 3 \\
                           t^2 & t \geq 3
                         \end{cases} \) in unit step function notation.

5. (10 points)  
   a. Make a simplified guess for a particular solution of \((D^2 - 2D + 1)x = te^t\). \textbf{You do not need to determine the coefficients!}  
   b. Find the general solution of \((D^2 - 2D + 1)x = e^t \sqrt{t}\) for \(t > 0\).

6. (10 points)  
   a. Compute \(\mathcal{L} [e^{5t+3}]\) using the definition. \textbf{No credit by any other method}  
   b. State for which values of \(s\) this Laplace transform is defined.

7. (10 points) Find \(1 \ast \cos 5t\).

8. (10 points) Find the Laplace transform of \(f(t) = te^{5t} \sin 3t\).

9. (20 points) Find the inverse Laplace transform of \(\frac{s + 3}{s^2 + 4s + 5}\).  
   a. \(\frac{s}{s^2 + 5}\)  
   b. \(\frac{1}{(s^2 + 5)^2}\).

\textit{Examination continues on next page}
10. (20 points) Solve using the Laplace transform. No credit by any other method.

a. \( x'' + 4x' + 4x = t^2 e^{-2t}, \quad x(0) = x'(0) = 0. \)

b. \((D - 1)x = \begin{cases} 
    t^2 & \text{if } t < 2 \\
    t^2 + 1 & \text{if } t \geq 2 
\end{cases}, \quad x(0) = 1. \) \( s - 1)\mathcal{L}[x] - 1 = \mathcal{L}[t^2] + \mathcal{L}[u_2(t)] = \frac{2}{s^3} + e^{-2s} \mathcal{L}[1] = \frac{2}{s^3} + \frac{e^{-2s}}{s}, \) so

\[
\mathcal{L}[x] = \frac{1}{s - 1} + \frac{2}{s^3(s - 1)} + \frac{e^{-2s}}{s(s - 1)}. 
\]

Now do partial fractions decompositions: setting

\[
\frac{1}{s^3(s - 1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{E}{s - 1}
\]
gives

\[
As^2(s - 1) + Bs(s - 1) + C(s - 1) + Es^3 = 1 \quad \text{or} \quad A + E = 0, \quad B - A = 0, \quad C - B = 0, \quad -C = 1,
\]
giving \( C = -1, \quad B = -1, \quad A = -1, \) and \( E = 1. \) Similarly, set

\[
\frac{1}{s(s - 1)} = \frac{A}{s} + \frac{B}{s - 1},
\]
giving \( A(s - 1) + Bs = 1, \) or \( A = -1, B = 1. \)

So,

\[
x(t) = \mathcal{L}^{-1} \left[ \frac{1}{s - 1} + \frac{2}{s^3(s - 1)} + \frac{e^{-2s}}{s(s - 1)} \right] \\
= e^t + 2 \mathcal{L}^{-1} \left[ \frac{-1}{s} - \frac{1}{s^2} - \frac{1}{s^3} + \frac{1}{s - 1} \right] + u_2(t) \mathcal{L}^{-1} \left[ \frac{-1}{s} + \frac{1}{s - 1} \right] (t - 2) \\
= e^t + 2 \left( -1 - t - \frac{t^2}{2} + e^t \right) + u_2(t) \left( -1 + e^{t-2} \right) \\
= 3e^t - 2 - 2t - t^2 - u_2(t) \left( 1 - e^{t-2} \right).
\]

END OF EXAMINATION