1. (30 points, 5 each) **No partial credit.**
   a. Check for independence of:
      1) the collection of functions $x$, $x \ln x$, $x \ln x^2$;
      2) the collection of vectors
         \[
         \begin{pmatrix}
         1 \\
         2 \\
         3 \\
         4
         \end{pmatrix},
         \begin{pmatrix}
         8 \\
         7 \\
         6 \\
         5
         \end{pmatrix},
         \begin{pmatrix}
         13 \\
         8 \\
         3 \\
         -2
         \end{pmatrix},
         \begin{pmatrix}
         12 \\
         10 \\
         4 \\
         -1
         \end{pmatrix};
         \]
   b. Use the definition of the Laplace transform to compute $L[te^{-t}]$;
   c. Find $(D^2 + 2D + 1)[e^{2t} \sin t]$;
   d. Evaluate $e^t * t$;
   e. Find all solutions of the equation
      \[x' + x = x^2;\]
   f. Solve the initial value problem
      \[x' + x = x^2, \quad x(0) = \frac{1}{4}.\]

2. (6 points) Find the general solution of the non-homogeneous equation
   \[x' + x \tan t = \sin 2t.\]

3. (10 points) Consider the following initial value problem:
   \[x' = \sqrt{|x|}, \quad x(0) = 0.\]
   a. Is the existence and uniqueness theorem applicable?
   b. If it is not applicable, does the IVP have a solution?
   c. If a solution exists, is it unique? Explain.

*Examination continues on other side*
4. (8 points) Find the inverse Laplace transform of
   \[ \frac{2s - 1}{s^2 - 4s + 8}; \]
   \[ \frac{e^{-s}}{s^3 + 4s^2 + 4s}. \]
   
5. (8 points) Let the matrix
   \[ A = \begin{pmatrix} 1 & 1 & 0 \\ -2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \]
   be given.
   
   a. \( A \) has an eigenvalue \( \lambda = 1 \). Find an eigenvector corresponding to this eigenvalue;
   
   b. The vector \( v = \begin{pmatrix} -1 \\ i \\ 1 \end{pmatrix} \) is an eigenvector of \( A \). Find the corresponding eigenvalue.

6. (10 points) Use the Laplace transform method to solve
   \[ (D^3 - D)x = \begin{cases} 1, & \text{if } t < 2, \\ 0, & \text{if } t \geq 2, \end{cases} \quad x(0) = x'(0) = x''(0) = 0. \]
   
   No credit for any other method.

7. (10 points) Solve the system of differential equations
   \[ D\vec{x} = \begin{pmatrix} 1 & -1 & -4 \\ 2 & -2 & -2 \\ 1 & 0 & -4 \end{pmatrix}\vec{x}. \]

8. (8 points) Solve the system of differential equations
   \[ D\vec{x} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\vec{x} + \begin{pmatrix} 2e^{-2t} \\ e^{-2t} \end{pmatrix}. \]

9. (10 points) Consider the system
   \[ \frac{dx}{dt} = -y, \]
   \[ \frac{dy}{dt} = x - 2y(1 + x^2). \]
   
   a. Show that \( E = x^2 + y^2 \) is a Lyapunov function for this system;
   
   b. Find equilibrium points;
   
   c. Classify each equilibrium;
   
   d. Find the linearized matrix for each equilibrium point;
   
   e. Draw the phase portrait of the linearization of each equilibrium;
   
   f. Determine whether the system has closed integral curves.