No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (36 points, 6 each) **These questions have no partial credit.**
   a. Check for independence.
   
   \[
   \begin{pmatrix}
   1 & 4 & 1 & 3 & 1 \\
   2 & 5 & 0 & -1 & 1 \\
   3 & 6 & 0 & 0 & 1 \\
   4 & 7 & 0 & 2 & 1 \\
   \end{pmatrix}
   \]

   b. Use the definition to compute \( \mathcal{L}[e^{-t}] \).

c. Find \( \mathcal{L}[te^{2t} \sin 3t] \).

d. Evaluate \( 1 * t \).

e. Find all solutions of the form \( x = e^{at} \) or \( x = t^a \) for the equation

   \[
   (t^2D^2 - tD)x = 0
   \]

   f. Solve the nonhomogeneous equation

   \[
   (t^2D^2 - tD)x = t^{-1} \quad t > 0
   \]

The questions below have partial credit.

2. (5 points) Solve \( tx' - x = t^3 \quad x(1) = 0 \)

3. (8 points)
   a. Show that for any \( b \geq 0 \)

   \[
   x(t) = \begin{cases} 
   0 & t \leq b \\
   (t - b)^5 & t > b 
   \end{cases}
   \]

   is a solution of

   \[
   (\ast) \quad \frac{dx}{dt} = 5x^{4/5} \quad x(0) = 0
   \]

   b. Does (\ast) have a unique solution?

c. Does this fact contradict the existence and uniqueness theorem? Explain why or why not.

*Examination continues on other side*
4. (5 points) Solve

\[
\begin{pmatrix}
1 & 2 & 1 & -1 & -1 \\
2 & 2 & 2 & -3 & -2 \\
-1 & 0 & -1 & 2 & 1
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{pmatrix}
= 
\begin{pmatrix}
2 \\
1 \\
0
\end{pmatrix}
\]

5. (10 points) Solve

\[
D \vec{x} = \begin{pmatrix}
2 & 0 & 4 \\
0 & 2 & 0 \\
1 & 0 & 2
\end{pmatrix} \vec{x}
\]

6. (8 points) Solve

\[
D \vec{x} = \begin{pmatrix}
0 & -3 & 2 \\
3 & 0 & -3 \\
0 & 0 & 2
\end{pmatrix} \vec{x} + \begin{pmatrix}
e^{2t} \\
3 \\
e^{2t}
\end{pmatrix}
\]

7. (8 points) Find the inverse Laplace transform of

a. \( \frac{s + 1}{s^2 + 6s + 9} \)

b. \( \frac{1}{s^3 + 6s^2 + 9s} \)

8. (10 points) Solve

\[
(D^2 + 1)x = \begin{cases}
sin t & t \leq \pi \\
0 & t > \pi
\end{cases}, \quad x(0) = x'(0) = 0
\]

9. (10 points) Consider the system

(S)

\[
\begin{align*}
\frac{dx}{dt} &= 10x - 6y \\
\frac{dy}{dt} &= -6x + 10y
\end{align*}
\]

a. Show that \( E = -5x^2 + 6xy - 5y^2 \) is a Lyapunov function for (S).

b. Determine whether (S) has closed integral curves.