9-22  **Ideal Rankine Cycle**

**Given:** Steam power plant with ideal Rankine cycle. Net power output is 45 MW. Steam enters the turbine at 7 MPa and 500 °C. The steam is cooled in the condenser at 10 kPa using cooling water at a rate of 2000 kg/s.

**Find:** a) thermal efficiency of the cycle, b) mass flow rate of the steam, and c) temperature rise of the cooling water. Draw the T-s diagram.

**Solution:**

At state 3, $P_3 = 7$ MPa and $T_3 = 500$ °C. We have superheated vapor: $h_3 = 3410.3$ kJ/kg and $s_3 = 6.7975$ kJ/kgK.

At state 4, $s_4 = s_3$ and $P_4 = 10$ kPa. At 10 kPa, $s_f = 0.6493$ kJ/kgK and $s_{fg} = 7.5009$ kJ/kgK → $x_4 = 0.8197$.

$h_f = 191.83$ kJ/kg and $h_{fg} = 2392.8$ kJ/kgK → $h_4 = 2153.12$ kJ/kg.

At state 1, we have saturated liquid at 10 kPa → $v_1 = v_f = 0.001010$ m$^3$/kg, $h_1 = h_f = 191.83$ kJ/kg.

The work consumed by the pump is: $w_{pump} = v_1(P_1 - P_2) = 0.001010(10 - 7000) = -7.0599$ kJ/kg.

State 2 is found by: $h_1 - h_2 = w_{pump} → h_2 = h_1 - w_{pump} = 191.83 - (-7.0599) = 198.89$ kJ/kg.

The work produced by the turbine is: $w_{turbine} = h_3 - h_4 = 3410.3 - 2153.12 = 1257.18$ kJ/kg.

a) The thermal efficiency is:

$$\eta_h = \frac{W_{net}}{Q_{in}} = \frac{W_{pump} + W_{turbine}}{h_3 - h_2} = \frac{-7.0599 + 1257.18}{3410.3 - 198.89} = \frac{1250.12}{3211.41} = 0.389, \text{ or } 38.9 \text{ percent.}$$

b) The mass flow rate is: $45$ MW/$w_{net} = 45,000/1250.12 = 35.99$ kg/s.

c) The heat given off in the condenser is: $Q = m(h_1 - h_4) = 35.99(191.83 - 215312) = -70.6$ MW.

This amount of heat is being absorbed by the cooling water:

$$Q_{cw} = m_{cw}c_p\Delta T → \Delta T = Q_{cw}/m_{cw}c_p = 70600/(2000 \cdot 4.18) = 8.445 \text{ °C}$$

The T-s diagram is shown on the right. Some of the features are:

- state 1 is saturated liquid
- state 4 is saturated mixture
- state 3 is superheated
- 1→2 and 3→4 are isentropic
- $P_2 = P_3$; $P_1 = P_4$. 
9-29 Reheat Rankine Cycle

**Given:** Steam power plant that operates on a reheat Rankine cycle with 80 MW of net output. Steam enters the high pressure turbine at 10 MPa and 500 °C, and the low pressure turbine at 1 MPa and 500 °C. Steam leaves the condenser as a saturated liquid at 10 kPa. Isentropic efficiencies of the turbine and compressor are 80 percent and 95 percent, respectively.

**Find:** a) quality or temperature of steam at turbine exit, b) thermal efficiency of the cycle, and c) mass flow rate of the steam. Draw the T-s diagram.

**Solution:**

**State 1** is saturated liquid at 10 kPa:

\[ h_1 = h_f = 191.83 \text{ kJ/kg}, \quad v_1 = v_f = 0.001010 \text{ m}^3/\text{kg}. \]

**State 2s** is at 10 MPa.

The ideal work of the pump is:

\[ w_{\text{pump}} = v_1 (P_1 - P_2) = h_1 - h_2 \]

\[ h_{2s} = h_1 - v_1 (P_1 - P_2) = 191.83 - 0.001010(10 - 10,000) = 201.92 \text{ kJ/kg}. \]

**State 2a** is found from the isentropic efficiency of the compressor:

\[ \eta_c = (h_{2s} - h_1)/(h_{2a} - h_1) \]

\[ h_{2a} = (h_{2s} - h_1)/\eta_c + h_1 = (201.92 - 191.83)/0.95 + 191.83 = 202.45 \text{ kJ/kg}. \]

**State 3** is at 10 MPa and 500 °C:

\[ h_3 = 3373.7 \text{ kJ/kg}, \quad s_3 = 6.5966 \text{ kJ/kgK}. \]

**State 4s** is at 1 MPa and \( s_4 = s_3 \):

interpolate between \( T_{\text{sat}} \) and 200 °C to get \( h_{4s} = 2782.78 \text{ kJ/kg}. \)

**State 4a** is found from the isentropic efficiency of turbine:

\[ h_{4a} = \eta_t (h_{4s} - h_3) + h_3 = 0.8(2782.78 - 3373.7) + 3373.7 = 2900.96 \text{ kJ/kg}. \]

**State 5** is at 1 MPa and 500 °C:

\[ h_5 = 3478.5 \text{ kJ/kg}, \quad s_5 = 7.7622 \text{ kJ/kgK}. \]

**State 6s** is at 10 kPa and \( s_6 = s_5 \):

This is saturated mixture. \( s_f = 0.6493 \text{ kJ/kgK}, \quad s_{fg} = 7.5009 \text{ kJ/kgK}. \)

\[ x_{6s} = (s_6 - s_f)/s_{fg} = (7.7622 - 0.6493)/7.5009 = 0.9483. \]

\[ h_f = 191.83 \text{ kJ/kg}, \quad h_{fg} = 2392.8 \text{ kJ/kg} \]

\[ h_{6s} = 2460.86 \text{ kJ/kg}. \]

**State 6a** is found from the isentropic efficiency of turbine:

\[ h_{6a} = \eta_t (h_{6s} - h_5) + h_5 = 0.8(2460.86 - 3478.5) + 3478.5 = 2664.38 \text{ kJ/kg}. \]

a) This is greater than \( h_g \), so we have a superheated vapor.

Interpolate between 50 °C and 100 °C to find \( T_6 = 87.82 \text{ °C} \).

b) The thermal efficiency of the reheat cycle is given by:

\[ \eta_h = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{h_1 - h_2 + h_3 - h_4 + h_5 - h_6}{h_3 - h_2 + h_5 - h_4} = \frac{191.83 - 202.45 + 3373.7 - 2900.96 + 3478.5 - 2664.38}{3373.7 - 201.92 + 3478.5 - 2900.96} \]

\[ = \frac{1276.24}{3749.32} = 0.3404, \text{ or } 34.04 \text{ percent}. \]

c) The mass flow rate is given by:

\[ 80 \text{ MW}/w_{\text{net}} = 80,000 /1276.24 = 62.68 \text{ kg/s}. \]

The important features of the T-s diagram are:

- state 1 is saturated liquid
- state 6s is saturated mixture
- states 3, 4s, 4a, 5, 6a are superheated
- \( T_3 = T_5; \quad P_2 = P_3; \quad P_4 = P_5; \quad P_1 = P_6. \)
9-67  Combined gas-steam power plant

Given: Combined gas-steam power plant with net power output of 450 MW. Gas turbine cycle has a pressure ratio of 14. Air enters the compressor at 300 K and the turbine at 1400 K. The combustion gases at turbine exhaust is used to heat the steam at 8 MPa to 400 °C in a heat exchanger. The combustion gases leave the heat exchanger at 460 K. An open feedwater heater in the steam cycle operates at 0.6 MPa. The condenser pressure is 20 kPa.

Find: a) mass flow rate ratio of air to steam; b) required heat input into the combustion chamber; c) thermal efficiency of the combined cycle.

Solution:
The air cycle is an ideal Brayton cycle. Using variable specific heats:

State 1: \( T_1 = 300 \text{ K} \rightarrow h_1 = 300.19 \text{ kJ/kg}, \ P_{r_1} = 1.3860 \)

State 2: \( P_{r_2} = P_{r_1} \left( \frac{P_2}{P_1} \right) = 1.3860 \times 14 = 19.404 \)

Interpolate between 620 K and 630 K \( \rightarrow h_2 = 635.52 \text{ kJ/kg} \).

State 3: \( T_3 = 1400 \text{ K} \rightarrow h_3 = 1515.42 \text{ kJ/kg}, \ P_{r_3} = 450.5 \)

State 4: \( P_{r_4} = P_{r_3} \left( \frac{P_4}{P_3} \right) = 450.5/14 = 32.179 \)

Interpolate between 720 K and 730 K \( \rightarrow h_4 = 735.83 \text{ kJ/kg} \).

State 1*: At the exhaust of the heat exchanger, \( T_{1*} = 460 \text{ K} \rightarrow h_{1*} = 462.02 \text{ kJ/kg} \).

The steam cycle is a regenerative Rankine cycle with open feedwater heater.

State 5 is a saturated liquid at 20 kPa:
\( h_5 = 251.40 \text{ kJ/kg}, v_5 = 0.001017 \text{ m}^3/\text{kg} \).

State 6 is at the feedwater pressure of 0.6 MPa.

\( w_{\text{pump,1}} = v_5 (P_5 - P_6) = 0.001017 (20 - 600) = -0.5899 \text{ kJ/kg} \).

\( h_6 = h_5 - w_{\text{pump,1}} = 251.40 - (-0.5899) = 251.99 \text{ kJ/kg} \).

State 7 is saturated liquid at 0.6 MPa:
\( h_7 = 670.56 \text{ kJ/kg}, v_7 = 0.001101 \text{ m}^3/\text{kg} \).

State 8 is at 8 MPa.

\( w_{\text{pump,2}} = v_7 (P_7 - P_8) = 0.001101 (600 - 8000) = -8.1474 \text{ kJ/kg} \).

\( h_8 = h_7 - w_{\text{pump,2}} = 670.56 - (-8.1474) = 678.71 \text{ kJ/kg} \).

State 9 is at 8 MPa and 400 °C: \( h_9 = 3138.3 \text{ kJ/kg}, s_9 = 6.3634 \text{ kJ/kgK} \).

State 10 is at 0.6 MPa and \( s_{10} = s_9: s_r = 1.9312 \text{ kJ/kgK} \) and \( s_{fr} = 4.8288 \text{ kJ/kgK} \) \( \rightarrow x_{10} = 0.9179 \).

\( h_10 = 670.56 \text{ kJ/kg}, h_{19} = 2086.3 \text{ kJ/kg} \rightarrow h_{11} = 2585.5 \text{ kJ/kg} \).

State 11 is at 20 kPa and \( s_{11} = s_9: s_r = 0.8320 \text{ kJ/kgK} \) and \( s_{fr} = 7.0766 \text{ kJ/kgK} \) \( \rightarrow x_{11} = 0.7816 \).

\( h_f = 251.40 \text{ kJ/kg}, h_{19} = 2358.3 \text{ kJ/kg} \rightarrow h_{11} = 2094.76 \text{ kJ/kg} \).

Heat balance around the open FWH to find \( y: y h_{10} + (1 - y) h_6 = h_7 \)
\[
y = \frac{h_7 - h_6}{h_{10} - h_6} = \frac{670.56 - 251.99}{2585.5 - 251.99} = 0.179.
\]
9-67 (continued)

a) The air-to-steam ratio is found from heat balance around the heat exchanger:

\[
\dot{m}_{\text{air}} (h_1 - h_4) = \dot{m}_{\text{steam}} (h_8 - h_9) \Rightarrow \frac{\dot{m}_{\text{air}}}{\dot{m}_{\text{steam}}} = \frac{h_8 - h_9}{h_1 - h_4} = \frac{678.71 - 3138.3}{462.02 - 735.83} = 8.98.
\]

b) We first need to find the mass flow rates.

The net work of the Brayton cycle is:

\[
W_{\text{net,B}} = h_1 - h_2 + h_3 - h_4 = 300.19 - 635.52 + 1515.42 - 735.83 = 444.26 \text{ kJ/kg of air}.
\]

The work from the Rankine cycle's turbine is:

\[
W_{\text{turbine,R}} = \gamma (h_9 - h_{10}) + (1 - \gamma)(h_9 - h_{11})
\]

\[
= 0.179(3138.3 - 2585.5) + (1 - 0.179)(3138.3 - 2094.76) = 955.70 \text{ kJ/kg of steam}.
\]

The net work from the Rankine cycle is:

\[
W_{\text{net,R}} = W_{\text{pump,1}} + W_{\text{pump,2}} + W_{\text{turbine,R}} = (-0.5899) + (-8.1474) + 955.70 = 946.96 \text{ kJ/kg}.
\]

The net work is:

\[
W_{\text{net}} = \dot{m}_{\text{air}} W_{\text{net,B}} + \dot{m}_{\text{steam}} W_{\text{net,R}} = 8.98\dot{m}_{\text{steam}} W_{\text{net,B}} + \dot{m}_{\text{steam}} W_{\text{net,R}}.
\]

Rearrange to solve for mass flow rates:

\[
\dot{m}_{\text{steam}} = \frac{W_{\text{net}}}{8.98 W_{\text{net,B}} + W_{\text{net,R}}} = \frac{450,000}{8.98(444.26) + 946.96} = 91.159 \text{ kg/s of steam}
\]

\[
\dot{m}_{\text{air}} = 8.98\dot{m}_{\text{steam}} = 818.61 \text{ kg/s of air}
\]

The required heat input is:

\[
Q_{\text{in}} = \dot{m}_{\text{air}} (h_3 - h_2) = 818.61(1515.42 - 635.52) = 720.3 \text{ MW}.
\]

\[
\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{450}{720.3} = 0.6247, \text{ or 62.5 percent}.
\]

9-84 Reheat-Regenerative Rankine Cycle

Given: Ideal reheat-regenerative Rankine cycle with one open feedwater heater. The boiler pressure is 10 MPa, condenser pressure is 15 kPa, re heater pressure is 1 MPa, and feedwater pressure is 0.6 MPa. Steam enters the high and low pressure turbines at 500 °C.

Find: a) fraction of steam extracted for regeneration (\(\gamma\)), and b) thermal efficiency of the cycle. Draw the T-s diagram.

Solution:

State 1 is saturated liquid at the condenser pressure of 15 kPa:

\[
h_1 = 225.94 \text{ kJ/kg}, v_1 = 0.001014 \text{ m}^3/\text{kg}.
\]

State 2 is at the feedwater pressure of 0.6 MPa:

\[
w_{\text{pump,1}} = v_1(P_1 - P_2) = 0.001014(15 - 600) = -0.5932 \text{ kJ/kg}.
\]

\[
h_2 = h_1 - w_{\text{pump,1}} = 225.94 - (-0.5932) = 226.53 \text{ kJ/kg}.
\]

State 3 is saturated liquid at 0.6 MPa:

\[
h_3 = 670.56 \text{ kJ/kg}, v_3 = 0.001101 \text{ m}^3/\text{kg}.
\]

State 4 is at the boiler pressure of 10 MPa:

\[
w_{\text{pump,2}} = v_3(P_3 - P_4) = 0.001101(600 - 10,000) = -10.349 \text{ kJ/kg}.
\]

\[
h_4 = h_3 - w_{\text{pump,2}} = 670.56 - (-10.349) = 680.91 \text{ kJ/kg}.
\]
9-84 (continued)

State 5 is at 10 MPa and 500 °C: \( h_5 = 3373.7 \text{ kJ/kg}, s_5 = 6.5966 \text{ kJ/kgK}. \)

State 6 is at the reheater pressure of 1 MPa and \( s_6 = s_5 \):
Interpolate between sat. temperature and 200 °C \( \Rightarrow h_6 = 2782.78 \text{ kJ/kg}. \)

State 7 is at 1 MPa and 500 °C: \( h_7 = 3478.5 \text{ kJ/kg}, s_7 = 7.7622 \text{ kJ/kgK}. \)

State 8 is at 0.6 MPa and \( s_8 = s_7 \): interpolate between 400 °C and 500 °C \( \Rightarrow h_8 = 3309.52 \text{ kJ/kg}. \)

State 9 is at 15 kPa and \( s_9 = s_7 \): \( x_9 = 0.966, h_9 = 2518.46 \text{ kJ/kg}. \)

a) The fraction of steam extracted for regeneration is found from a heat balance around the open feedwater heater:

\[
y h_8 + (1 - y) h_2 = h_3 \\
y = \frac{h_3 - h_2}{h_9 - h_2} = \frac{670.56 - 226.53}{3309.52 - 226.53} = 0.144.
\]

b) The thermal efficiency of the cycle is found from \( \frac{w_{\text{net}}}{q_{\text{in}}} \).

The work for the high-pressure turbine is: \( w_{\text{turbine},1} = h_5 - h_6 = 3373.7 - 2782.78 = 590.92 \text{ kJ/kg}. \)

The work for the low-pressure turbine is:

\[
w_{\text{turbine},2} = y(h_7 - h_6) + (1 - y)(h_7 - h_8) = 0.144(3478.5 - 3309.52) + (1 - 0.144)(3478.5 - 2518.46) = 846.13 \text{ kJ/kg}. \]

The net work is:

\[
w_{\text{turbine},1} + w_{\text{turbine},2} + w_{\text{pump},1} + w_{\text{pump},2} = 590.92 + 846.13 + 10.349 = 1426.4 \text{ kJ/kg}. \]

The heat input is: \( q_{\text{in}} = h_5 - h_4 + h_7 - h_6 = 3373.7 - 680.91 + 3478.5 - 2782.78 = 3388.51 \text{ kJ/kg}. \)

The thermal efficiency is: \( \frac{w_{\text{net}}}{q_{\text{in}}} = 1423.1/3388.51 = 0.4209, \text{ or 42.1 percent}. \)

The T-s diagram should have the following features:
- states 1 and 3 are saturated liquids
- states 5, 6, 7, 8 are superheated vapor
- state 9 is saturated mixture
- \( P_1 = P_9; P_2 = P_3 = P_6; P_4 = P_5; P_6 = P_7 \)
- \( 1 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, \text{ and } 7 \rightarrow 8 \rightarrow 9 \) are isentropic
- states 5 and 7 have the same temperature
10-11 Ideal Refrigeration Cycle

Given: An ideal vapor-compression refrigeration cycle uses R-134a between 0.12 and 0.7 MPa. The mass flow rate is 0.05 kg/s.

Find: a) Rate of heat removal from refrigerated space and the power input into the compressor; b) rate of heat rejection to the environment; and c) coefficient of performance. Draw the T-s diagram.

Solution:
State 1 is saturated vapor at 0.12 MPa
\[ h_1 = h_f = 233.86 \text{ kJ/kg}, \quad s_1 = s_f = 0.9354 \text{ kJ/kgK} \]

State 2 is at 0.7 MPa, and \( s_2 = s_1 \). Interpolate between 30 °C and 40 °C to get
\[ h_2 = 270.22 \text{ kJ/kg} \]

State 3 is saturated liquid at 0.7 MPa.
\[ h_3 = h_f = 86.78 \text{ kJ/kg} \]

State 4 is at 0.12 MPa, and \( h_4 = h_3 \).

a) The heat removal from refrigerated space is between states 4 and 1:
\[ Q_{in} = m(h_4 - h_1) = 0.05(233.86 - 86.78) = 7.354 \text{ kW} \]

The power input to the compressor is between states 1 and 2:
\[ W_{in} = m(h_1 - h_2) = 0.05(233.86 - 270.22) = -1.818 \text{ kW}, \text{ or } 1.818 \text{ kW} \text{ of power input.} \]

b) Heat is rejected between states 2 and 3:
\[ Q_{out} = m(h_3 - h_2) = 0.05(86.8 - 270.22) = -9.171 \text{ kW}, \text{ or } 9.171 \text{ kW} \text{ of heat rejected.} \]

c) COP = \( \frac{Q_{in}}{W_{in}} = 7.354/1.818 = 4.045 \)

The T-s diagram is characterized by:
- state 1 is saturated vapor
- state 2 is superheated vapor
- state 3 is saturated liquid
- state 4 is saturated mixture
- \( P_2 = P_3; \quad P_4 = P_1 \)
- \( 1\rightarrow 2 \) is isentropic
10-19  Non-Ideal Refrigeration Cycle

Given:  R-134a enters the compressor of a refrigeration cycle at 140 kPa and -10 °C at a rate of 0.3 m³/min, and leaves at 1 MPa. The isentropic efficiency of the compressor is 78 percent. The refrigerant enters the throttling valve at 0.95 MPa and 30 °C, and leaves the evaporator as saturated vapor at -18.5 °C.

Find:  a) Power input to the compressor; b) rate of heat removal from the refrigerated space; and c) pressure drop and the rate of heat gained in the line between evaporator and compressor. Draw the T-s diagram.

Solution:

State 1 is superheated at 140 kPa and -10 °C:  h₁ = 243.40 kJ/kg,  s₁ = 0.9606 kJ/kgK,  v₁ = 0.14549 m³/kg.

The mass flow rate is:  \( \dot{m} = \frac{V}{v_1} = \frac{0.3 \text{ m}^3 \text{ min}^{-1}}{60 \text{ sec}} \cdot \frac{1}{0.14549} = 0.03437 \text{ kg/s} \)

State 2s is at 1 MPa, and  s₂ = s₁. Interpolate between 50 °C and 60 °C to get  h₂s = 286.04 kJ/kg.

State 2 can be found from the isentropic efficiency of the compressor.

\[ \eta_k = \frac{(h_{2s} - h_1)}{(h_{2a} - h_1)} \]

\[ h_{2a} = \frac{(h_{2s} - h_1)}{\eta_k} + h_1 = \frac{286.04 - 243.40}{0.78} + 243.40 = 298.07 \text{ kJ/kg} \]

State 3 is at 0.95 MPa and 30 °C, which is a compressed liquid. We will use the properties of saturated liquid at 30 °C:  h₃ = 91.49 kJ/kg.

State 4 is a saturated mixture at -18.5 °C, and  h₄ = h₃.

State 1* is a saturated vapor at -18.5 °C.

We can interpolate between -20 °C and -18 °C to find  h₁* = 236.23 kJ/kg and  P₁* = P_sat = 141.87 kPa.

a) The power input to the compressor is between states 1 and 2:

\[ W_{in} = \dot{m}(h_1 - h_{2a}) = 0.03437(243.40 - 298.07) = -1.879 \text{ kW, or 1.879 kW} \]

b) The heat removal from refrigerated space is between states 4 and 1*:

\[ Q_{in} = \dot{m}(h_4 - h_{1*}) = 0.03437(236.23 - 91.49) = 4.975 \text{ kW} \]

c) The pressure drop between states 1* and 1 is: 141.87 - 140 = 1.87 kPa.

The rate of heat gained is:  \( Q = \dot{m}(h_1 - h_{1*}) = 0.03437(243.40 - 236.23) = 0.246 \text{ kW} \)

The T-s diagram is characterized by:

- state 1* is saturated vapor
- states 1, 2s, and 2a are superheated vapor
- state 3 is compressed liquid
- state 4 is saturated mixture
- \( P_2 = P_3; \ P_4 = P_1* \)
- 1→2s is isentropic
10-38  Cascade Refrigeration Cycle

Given:  Two-stage cascade refrigeration system operating between 0.8 MPa and 0.14 MPa. Each stage is an ideal vapor-compression cycle with R-134a. Heat rejection from the lower cycle to the upper cycle takes place in an adiabatic heat exchanger where both streams enter at 0.4 MPa. The mass flow rate of the refrigerant through the upper cycle is 0.24 kg/s.

Find:  a) mass flow rate of the refrigerant through lower cycle; b) rate of heat removal from the refrigerated space and the power input to the compressor; c) coefficient of performance.

Solution:

**Lower Cycle:**

State 1 is a saturated vapor at 0.14 MPa
\( h_1 = 236.04 \text{ kJ/kg}, \quad s_1 = 0.9322 \text{ kJ/kgK}. \)

State 2 is at 0.4 MPa and \( s_2 = s_1. \) Interpolate between 10 °C and 20 °C to get \( h_2 = 257.39 \text{ kJ/kg}. \)

State 3 is a saturated liquid at 0.4 MPa \( \rightarrow h_3 = 62.00 \text{ kJ/kg}. \)

State 4 is a saturated mixture at 0.14 MPa, and \( h_4 = h_3. \)

**Upper Cycle:**

State 5 is a saturated vapor at 0.4 MPa \( \rightarrow h_5 = 252.32 \text{ kJ/kg}, \quad s_5 = 0.9145 \text{ kJ/kgK}. \)

State 6 is at 0.8 MPa and \( s_6 = s_5. \) Interpolate between \( T_{\text{sat}} \) and 40 °C to get \( h_6 = 266.59 \text{ kJ/kg}. \)

State 7 is a saturated liquid at 0.8 MPa \( \rightarrow h_7 = 93.42 \text{ kJ/kg}. \)

State 8 is a saturated mixture at 0.4 MPa, and \( h_8 = h_4. \)

a) The mass flow rate of the lower cycle is found from a heat balance around the heat exchanger:
\[
\dot{m}_L = \dot{m}_U \frac{h_5 - h_8}{h_2 - h_3} = 0.24 \frac{252.32 - 93.42}{257.39 - 62.00} = 0.195 \text{ kg/s.}
\]

b) The heat removal from refrigerated space is between states 4 and 1:
\[
Q_{in} = \dot{m}_L (h_4 - h_1) = 0.195(236.04 - 62.00) = 33.97 \text{ kW.}
\]

The total power input to the compressors is:
\[
W_{in} = \dot{m}_L (h_1 - h_2) + \dot{m}_U (h_3 - h_6)
\]
\[
= 0.195(236.04 - 257.39) + 0.24(252.32 - 266.59) = -4.163 + 3.425 = -0.738 \text{ kW, or 7.588 kW power input.}
\]

c) The coefficient of performance is: \( \text{COP} = \frac{Q_{in}}{W_{in}} = 33.97/7.588 = 4.477. \)
10-86 Heat Pump

Given: A heat pump operates on an ideal vapor-compression cycle with R-134a. The mass flow rate is 0.24 kg/s. The condenser and evaporator pressures are 900 kPa and 240 kPa, respectively.

Find: a) rate of heat supplied to the house; b) volume flow of the refrigerant at the compressor inlet; and c) coefficient of performance of the heat pump. Show the T-s diagram.

Solution:

State 1 is a saturated vapor at 240 kPa
\[ h_1 = 244.09 \text{ kJ/kg}, \ s_1 = 0.9222 \text{ kJ/kgK}, \ v_1 = 0.0834 \text{ m}^3/\text{kg}. \]

State 2 is at 900 kPa and \( s_2 = s_1 \). Interpolate between 40 °C and 50 °C to get \( h_2 = 271.41 \text{ kJ/kg} \).

State 3 is a saturated liquid at 900 kPa \( h_3 = 99.56 \text{ kJ/kg} \).

State 4 is a saturated mixture at 240 kPa, and \( h_4 = h_3 \).

a) The heat supplied to the house comes from the condenser, \( 2 \rightarrow 3 \):
\[ Q_{\text{out}} = m(h_3 - h_2) = 0.24(99.56 - 271.41) = -41.243 \text{ kW}, \text{ or } 41.24 \text{ kW} \text{ supplied to the house}. \]

b) The volume flow rate of the refrigerant at the compressor inlet (state 1) is:
\[ V = m v_1 = 0.24(0.0834) = 0.020 \text{ m}^3/\text{s}. \]

c) The coefficient of performance is:
\[ \text{COP} = \frac{Q_{\text{out}}}{W_{\text{in}}} \]
\[ W_{\text{in}} = m(h_1 - h_2) = 0.24(244.09 - 271.41) = -6.557 \text{ kW}, \text{ or } 6.557 \text{ kW power input}. \]
\[ \text{COP} = 41.24/6.557 = 6.290. \]

The T-s diagram has the following features:
- state 1 is saturated vapor
- state 2 is superheated vapor
- state 3 is saturated liquid
- \( 1 \rightarrow 2 \) is isentropic
- \( P_1 = P_4; \ P_2 = P_3 \).