Electrically Coupled Integrate-and-Fire Neurons
Erin Munro, Christoph Börgers
Department of Mathematics, Tufts University, Medford, MA 02155

Problem
Networks of neurons coupled by gap junctions may models high-frequency oscillations observed with epilepsy.

The integrate-and-fire model has proven to be very useful for modeling networks of neurons connected by synapses. However, it is much less common and easier to analyze than the Hodgkin-Huxley model.

The size and shape of the voltage spike is critical for determining how much current flows through the gap junction. However, the integrate-and-fire model does not model the spike explicitly, so the assumption that the model accurately models a network of neurons connected by gap junctions.

Methods

Hodgkin-Huxley Model

Voltage Spike
Spiking Current

Background

The Hodgkin-Huxley model is the gold-standard of neuronal models. In its original form, it is a set of 4-differential equations that describe the voltage inside the neuron. Here we use a 2-dimensional Hodgkin-Huxley model. In this model, the voltage v is at rest at 46 mV. If the neuron is stimulated, i.e. charge is injected, the voltage goes up if 

The voltage rises rapidly from an initial value v_0 to a voltage b of approximately 60 mV. Then it is gradually decreases back to the resting voltage v_0.

The maximum-differential equation is shown below. The other equations describe the changes of v_0 and v_0 to v_0, the same way the voltage v rises rapidly in the voltage shape.

Threshold:

At the end of the spike, the membrane returns to the resting potential (v_0). This graph shows a typical voltage and spiking current.

Results

For each model, we plotted the results for network 1 through 10. The graphs are in the left side show the spike times for each neuron. The graph on the right show the voltage trace of each neuron for network 1.

Discussion

The integrate-and-fire model with a fixed spiking current reproduces the Hodgkin-Huxley spikes better than the integrate-and-fire model with a fixed voltage spike. Some insight into why this is may be gained by comparing the voltage at the peak and the minimum from network 1 to the peak and the minimum from network 3. We can see that the peak voltage of the Hodgkin-Huxley model is 90 mV, while the peak voltage of the integrate-and-fire model is 90 mV. However, the integrate-and-fire model with a fixed voltage spike reproduces the Hodgkin-Huxley model more accurately.

We may modify the integrate-and-fire model by improving a fixed voltage spike as a fixed spiking current. To compare these techniques, we simulated 3 different neuronal networks using a Hodgkin-Huxley model, estimating the minimum current values. We chose a 4-voltage membrane potential of 0 mV to 100 mV. In network 2, we simulate the voltage and subthreshold value of network 2 at a fixed membrane 20 mV. We then simulate the propagation speed of the Hodgkin-Huxley model.

Networks

The current minimums of network 1 and 3 and the top minima of network 2 are mimicked 20 mV.

Bibliography/Acknowledgements


This work was supported by NIH grant R01NS041832. Many thanks to everyone who helped me write this paper!