Multimarket Contact, Price Discrimination and Consumer Welfare

George Norman
Department of Economics, Tufts University

Abstract
This paper investigates the extent to which firms choose multimarket contact when there is no possibility of consumer or producer arbitrage across the relevant markets and in the absence of the possibility of future cooperation. When firms employ uniform pricing a single-market firm enjoys a profit externality when in competition with a multi-market rival. As a result, the degree of multimarket contact is shown to be greater when firms offer products with distinctive characteristics and when setup costs are low. The ability of firms to price discriminate across markets removes the profit externality and increases multimarket contact, benefiting consumers but harming firms. There are prisoners' dilemma aspects to location choice. As a result, there are cases in which a collusive agreement by firms not to invade each others' markets is desirable and is sustainable by a trigger strategy based upon an easily observable deviation.

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George Norman
Department of Economics, Tufts University, Medford, MA 02155
1 Introduction

It is widely accepted that multimarket contact between non-cooperative firms is a useful mechanism by which to sustain tacitly (or explicitly) cooperative prices. Bernheim and Whinston (1990) provide the seminal contribution. Their work has been extended by, for example, Phillips and Mason (1996), Spagnolo (1999) and Matsushima (2001). It has also encouraged extensive testing and experimentation: see, for example, Evans and Kessides (1994), Parker and Röller (1997), Jans and Rosenbaum (1997), Busse (2000).

This analysis (and testing) leaves unresolved an important issue that provides the motivation for our analysis. Under what circumstances will firms choose strategies that lead to multimarket contact with potential rivals in the absence of the possibility of future cooperation? After all, as MacLeod, Norman and Thisse note:

“For collusion to be feasible, it is at least necessary that the participants are known to each other: an open-ended agreement would be difficult, if not impossible, to formulate.” (1987, p. 189).

The specific context that we consider is one in which non-cooperative firms choose whether or not to be active in particular markets where there is no possibility of consumer or producer arbitrage across these markets. Applications of this type of model include whether to establish operations in particular cities, regions or countries where transport and other transfer costs prevent firms from exporting from one city, region or country to another and prevent consumers from buying in other than their local market: retail, restaurant and telecommunications services are obvious examples. In the international context, our analysis determines whether a firm producing a non-tradeable good - retail, restaurant, consulting, banking, telecommunications and other services, for example - will adopt foreign direct investment, an issue that has received almost no

1 A possible exception is the idea, first suggested by Bernheim and Whinston (op. cit.), that a merger might be pursued because it increases multimarket contact with a competitor.
attention in the literature on foreign direct investment. Other applications would include the choice of time slots, again where rms and consumers do not arbitrage across time. Our analysis applies, for example, to cases where consumers prefer to eat either at lunchtime or in the evening but do not consider these different times to be substitutes.

As in much of the literature on product and location choice, an important determinant of the degree of multimarket contact that a rm chooses is whether or not the rm is able to price discriminate across the different markets. We do not model this choice explicitly, assuming rather that the choice of whether or not to price discriminate is chosen exogenously by rms or forced by the regulatory environment. What emerges clearly from our analysis, however, is that the ability to price discriminate may actually harm rms’ pro ts and bene t consumers.

The intuition is simple enough to explain. Consider two rms, one operating in two markets and the other in only one. With a commitment to uniform pricing in both markets, the multimarket rm has an incentive to maintain high prices in the more competitive market because any price reduction below the monopoly price in that market reduces its pro ts in its captive market. The rival rm bene ts in the more competitive market from the softer price competition in that market. This creates an incentive for the rival rm not to enter the captive market. No such incentive exists with discriminatory pricing, leading to a tougher competitive environment. A switch in the regime from uniform to discriminatory pricing, in other words, acts in a manner similar to the type of “shock” to the system considered by Bulow et al (1985). An obvious implication is that rms have incentives to nd ways of ensuring that they are unable to price discriminate, for example by advertising nationwide prices, while policy makers have incentives to nd ways in which price discrimination becomes more widely implemented.

Our analysis indicates that there are prisoners’ dilemma aspects to equilibria in which rms

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2 Graham (2001) is one exception in his analysis of foreign direct investment in the telecommunications sector.

3 An area in which our analysis has more limited applicability is in the choice of departure times - see, for example, Borenstein and Netz (1999) - since the “no arbitrage” condition is less likely to apply in its extreme form in such markets.

4 Thisse and Vives (1988) provide such an analysis in a different context.
choose location configurations with extensive multimarket contact. This provides an obvious justification for the suggestion that multimarket contact can lead to tacit cooperation in prices. If ...nd that multimarket contact reduces their pro...ts, they might well be expected to seek for ways in which to restore these pro...ts. Note, however, that in these circumstances multimarket contact emerges naturally as an equilibrium rather than being chosen strategically to facilitate coordination. Moreover, this same analysis suggests that ...rms will seek nonprice forms of tacit, or explicit collusion based on mutual forbearance in which they agree not to invade each other’s markets, that is, in which they actively seek to avoid multimarket contact. An obvious advantage of this type of tacitly collusive agreement as compared to agreements on price-setting is that the relevant punishment strategy is triggered by a deviation that is easy to detect. Two recent examples illustrate the point. First, the Competition Directorate of the European Union has successfully prosecuted a group of cement manufacturers accused of being part of a cartel, an important element of the evidence being the lack of market interpenetration by the relevant companies. Second, the Department of Justice signed a consent decree in January 2003 with New Times Media and Village Voice Media. These are newsweekly chains that were accused of dividing the markets in Cleveland and Los Angeles by closing competing newspapers:

“New Times Media agreed to close the New Times Los Angeles ... that competed with Village Voice’s L.A. Weekly. (A)t the same time Village Voice Media ... shut down the Cleveland Free Times, which shared a market with New Times Media’s Cleveland Scene.” (New York Times, January 27, 2003)

The remainder of the paper is structured as follows. In the next section we present a model of multimarket contact and in sections 3 and 4 respectively we identify the subgame perfect equilibria for this model with and without price discrimination. In section 5 we illustrate our analysis using a specific linear example. Section 6 briefly discusses some potential extensions of the analysis and our conclusions are presented in section 7.
2 A Model of Multimarket Contact

Assume that there are two non-cooperative firms, 1 and 2, each of which can supply a single differentiated product, 1 and 2 respectively, to two markets, a and b. These firms compete in a two-stage game. In the first stage the firms simultaneously choose which, if any, of the two markets to supply and in the second stage they compete in prices. We seek for a subgame perfect equilibrium to this game.

The markets might, for example, be two regions (the US and EU), two cities (Boston and Philadelphia) or two time slots (noon and evening). The essential features of these markets are, first, that for firm i to be able to supply product i to market k it must establish a local facility in k and second, that consumers in one market cannot purchase in the other market. The firms are assumed to have constant marginal costs $c$ in supplying either market and to incur fixed set-up costs $A$ in order to establish a facility in market $k$.

Define a location configuration as a pair $l = (l_1, l_2)$ where $l_1, l_2 \in \{a, b, ab\}$ and define $l$ as the set of feasible location configurations. Consumers are assumed to be identical in the two markets, with demand for product $i$ in market $k$ being given by

$$q_k = q_i p_{ik}^c l_{ik}^\pm (i; j = 1, 2; i \neq j; k = a, b; l \in l)$$

(1)

if both firms are active in market $k$ and

$$q_k = q_i (p_k) (i = 1, 2; k = a, b)$$

(2)

if only firm $i$ is active in $k$. In (1) $\pm$ is an inverse measure of the degree of product differentiation between the products offered by the two firms, which we normalize without loss of generality to $\pm 2 [0; 1]$. If $\pm = 0$ the products are totally differentiated, so that $q_i p_{ik}^c l_{ik}^\pm = q_i (p_k)$. By contrast, if $\pm = 1$ the products are identical. We make the normal assumptions that $\forall i, p_{ik}^c l_{ik}^\pm = p_{ik}^c < 0$, $\forall i, p_{ik}^c l_{ik}^\pm = p_{ik}^c > 0$, $\forall i, p_{ik}^c l_{ik}^\pm = p_{ik}^c < 0$ and $\forall i, p_{ik}^c l_{ik}^\pm = p_{ik}^c > 0$. In other words, the products are substitutes.
with the own-price effect being greater than the cross-price effect, and demand for both products decreases as the degree of product differentiation decreases.

We consider two cases. In the first, a rm is constrained to charge the same, uniform price in the markets in which it is active while in the second no such constraint applies.

3 Uniform Pricing

In order to construct the pay-off matrix for the first stage location game we need to consider the solution to the second-stage price subgame for each feasible location configuration.

3.1 \(l = \{a; 0g; f b; 0g; f 0; 0g; f 0; bg; f ab; 0g; f 0; abg\}\)

In each of these configurations only one rm is active in a market. Consider the location configuration \(a; 0g\). \(^5\) Firm 1 solves

\[
\max_{p_1} (p_1 - c)q(p_1) = \hat{\Delta}
\]

The solution is, of course, the monopoly price \(p^m\) satisfying the first-order condition:

\[
(p_1 - c)\hat{q} = \hat{\Delta} p_1 + q(p_1) = 0
\]

and profit is the monopoly profit \(\hat{\Pi} = (p^m - c)q(p^m) = \hat{\Delta}\). By the same argument, in the location configurations \(ab; 0g\) or \(f 0; abg\) the active rm is \(2(\hat{\Pi} \hat{\Delta})\). We make the reasonable assumption that \(\hat{\Delta} < \hat{\Pi}\) so that at least one rm is active in any SPE.

3.2 \(l = \{f a; ag; f b; bg; f ab; abg\}\)

In the location configuration \(f a; ag\) rm i solves:

\[
\max_{p_i} (p_i - c)q(p_i; p_j : \pm) = \hat{\Delta}
\]

\(^5\) In these and subsequent cases we need only consider one location configuration since the results apply to all permutations of that configuration.
This gives the pair of best response functions

\[ R_1: (p_1, c) \frac{\partial q(p_1; p_2; \pm)}{\partial p_1} + q(p_1; p_2; \pm) = 0 \]  

(6a)

\[ R_2: (p_2, c) \frac{\partial q(p_2; p_1; \pm)}{\partial p_2} + q(p_2; p_1; \pm) = 0 \]  

(6b)

We make the standard assumption:

Assumption 1: \( \frac{\partial^2 q(p_i; p_j; \pm)}{\partial p_i \partial p_j} - \frac{\partial^2 q(p_j; p_i; \pm)}{\partial p_j \partial p_i} > 0 \) for \( i \neq j \).

This guarantees that the slopes of the best response functions (6) satisfy \( 0 < \frac{d}{dp_i} = \frac{d}{dp_j} < 1 \).

The solution to (6) is, of course, the symmetric Bertrand duopoly price \( p^d(\pm) \) with profit to both firms:

\[ \frac{\pi^d(\pm)}{\pm} = \frac{\partial^2 q(p_i; p_j; \pm)}{\partial p_i \partial p_j} - \frac{\partial^2 q(p_j; p_i; \pm)}{\partial p_j \partial p_i} > 0 \]  

(7)

Standard analysis gives \( p^d(0) = p^m \) and \( p^d(1) = c \). It is reasonable to expect that \( \frac{d}{dp_i} = \frac{d}{dp_j} < 0 \) so that \( p^d(\pm) < p^m \) for \( \pm > 0 \). From (6) we then have:

\[ \frac{\partial^2 (\pm)}{\partial \pm} = i \frac{\partial^2 q(p_i; p_j; \pm)}{\partial p_i \partial p_j} - \frac{\partial^2 q(p_j; p_i; \pm)}{\partial p_j \partial p_i} < 0 \]  

(8)

As we would expect, profit to both firms in this location configuration is increasing in the degree of product differentiation. It also follows, of course, that \( \frac{\partial^2 (\pm)}{\partial \pm} < 0 \) for \( \pm > 0 \): a duopolist does not earn as much profit as a monopolist.

In the location configuration \( f_{ab}; ab \) the no-price discrimination constraint implies that firm 1 solves:

\[ \max_{p_1} 2 ((p_1, c) q(p_1; p_1, \pm) \frac{\partial q(p_1; p_1, \pm)}{\partial p_1}) \]  

(9)

In other words, the equilibrium prices are given by the solution to (6) with profits to each firm

\[ 2 \frac{\partial^2 (\pm)}{\partial \pm} \frac{\partial q(p_1; p_2; \pm)}{\partial p_1} > 0 \]

3.3 \( l = f_{ab}; ag; f_{ab}; bg; fa; abg; f_{b}; abg \)

Consider the location configuration \( f_{ab}; ag \). Firm 1 solves:

\[ \max_{p_1} (p_1, c) (q(p_1) + q(p_2; p_2; \pm)) \frac{\partial q(p_1)}{\partial p_1} \]  

(10)
while \(. \text{rm} \ 2 \) solves

\[
\max_{p_2} (p_2 \ i \ c) q(p_2; p_1 : \pm) \ i \ \hat{A} \tag{11}
\]

The best response functions are:

\[
R_1 : (p_1 \ i \ c) \frac{\partial q(p_1; p_2 : \mp)}{\partial p_1} + \frac{\partial q(p_1)}{\partial p_1} + q(p_1; p_2 : \pm) + q(p_1) = 0 \tag{12a}
\]

\[
R_2 : (p_2 \ i \ c) \frac{\partial q(p_1; p_2 : \mp)}{\partial p_2} + q(p_2; p_1 : \pm) = 0 \tag{12b}
\]

Denote the solution to (12) as \( p_{1f}^{ab;ag} (\pm) \); \( p_{2f}^{ab;ag} (\pm) \). It follows immediately that:

**Lemma 1** In the location configuration \( f_{ab;ag} \) the equilibrium to the second-stage price subgame with uniform pricing is such that

\[
\begin{align*}
(1) & \quad p^m > p_{1f}^{ab;ag} (\pm) > p_{2f}^{ab;ag} (\pm) > p^d (\pm) \text{ if } \pm \in (0;1) \\
(2) & \quad p_{1f}^{ab;ag} (0) = p_{2f}^{ab;ag} (0) = p^m \\
(3) & \quad p_{1f}^{ab;ag} (1) = p_{2f}^{ab;ag} (1) = c.
\end{align*}
\]

**Proof:** (ii) and (iii) are obvious. For (i), suppose that \( p_{1f}^{ab;ag} (\pm) = p_{2f}^{ab;ag} (\pm) = p^d (\pm) \). Then from (6) we have that (12b) is satisfied. However, the left-hand-side of (12a) is

\[
(p_1 \ i \ c) \frac{\partial q(p_1)}{\partial p_1} + q(p_1) \tag{13}
\]

which from (4) and \( p^d (\pm) < p^m \) is greater than zero for \( p_1 = p^d (\pm) \). It follows that the solution to (12) has \( p_1 > p^d (\pm) \). From (13) we know that the best response function (12a) lies to the right of (6a). It follows from Assumption 1 that \( p_{1f}^{ab;ag} (\pm) > p_{2f}^{ab;ag} (\pm) \). Assumption 1 also ensures that \( (p_2 \ i \ c) \frac{\partial q(p_2; p_1 : \pm)}{\partial p_2} + q(p_2; p_1 : \pm) > 0 \) for \( p_2 = p^d (\pm) \) and \( p_1 > p^d (\pm) \):

Now suppose that \( p_{2f}^{ab;ag} (\pm) = p_{2f}^{ab;ag} (\pm) = p^m \). Then from (4) and (6) we have that the left-hand side of (12a) is

\[
(p^m \ i \ c) \frac{\partial q(p^m; p^m : \pm)}{\partial p_1} + q(p^m; p^m : \pm) < 0 \tag{14}
\]

with the result that both \( \text{rms} \) want to charge a lower price.

The intuition for Lemma 1 is simple to explain. In the location configuration \( f_{ab;ag} \) the desire to compete in market \( a \) makes \( \text{rm} \ 1 \) want to charge less than the monopoly price. But
...rm 1 also recognizes that any cut in price in market a results in a loss of profits in its monopoly market b. This softens rm 1's willingness to reduce price in market a as compared to the location configurations fa; ag or fb; bg.

In the asymmetric location configuration fab; ag and its permutations, profit in the monopoly market is

\[
\frac{1}{4}^{\text{u}} (\pm) \hat{A} = \frac{3}{4} p_{1b}^{\text{fab; ag}} (\pm) i c q p_{1b}^{\text{fab; ag}} (\pm) i \hat{A} \tag{15}
\]

In the duopoly market profit to the multimarket rm is

\[
\frac{1}{4}^{\text{m}} (\pm) \hat{A} = \frac{3}{4} p_{2a}^{\text{fab; ag}} (\pm) i c q p_{2b}^{\text{fab; ag}} (\pm) \hat{A} \tag{16}
\]

and to the single market rm is

\[
\frac{1}{4}^{\text{s}} (\pm) \hat{A} = \frac{3}{4} p_{2a}^{\text{fab; ag}} (\pm) i c q p_{2b}^{\text{fab; ag}} (\pm) \hat{A} \tag{17}
\]

In all three cases the superscript u denotes uniform pricing.

We then have:

Lemma 2 In the asymmetric location configurations fab; ag and its permutations and with uniform pricing

(i) \( \frac{1}{4}^{\text{u}} (\pm) < \frac{1}{4} \)

(ii) \( \frac{1}{4}^{\text{u}} (\pm) > \frac{1}{4} (\pm) \)

(iii) \( \frac{1}{4}^{\text{u}} (\pm) < \frac{1}{4}^{\text{s}} (\pm) \)

Proof: (i) follows immediately from the proof of Lemma ???. To see (ii), note that rm 2's profit is increasing as we move up its best response function. (ii) then follows immediately from Lemma ???. Now consider (iii) and consider the two rns' best response functions. By symmetry of the demand functions, we know that if we start at \( p_1 = p^d (\pm) \) and \( p_2 = p^d (\pm) \) and move along a locus of slope unity in \([p_1; p_2]\) space the two rm's profits change identically. But the switch from fa; ag to fab; ag moves the equilibrium along the locus \( R_2 \) which has a slope less than unity from Assumption 1. Again by symmetry of the demand functions, this moves rm 2 to a higher isoprofit curve than rm 1.
The intuition is just that noted above. The uniform pricing constraint harms the multimarket rm in its captive market and benefits the rival rm in the competitive market. The multimarket rm may also benefit from the softer price competition in the competitive market but the rival always gains more from the externality created by uniform pricing. We would like to be able to claim that \( \frac{1}{2} \mu_m(\pm) < \frac{1}{2} \mu(\pm) \) but cannot do so unambiguously. If \( R_m^1 \) lies "sufficiently close" to \( R_1 \) then we could have \( \frac{1}{2} \mu_m(\pm) > \frac{1}{2} \mu(\pm) \). Fortunately, no such unambiguous relationship is necessary for the determination of the equilibrium location configurations.

### 3.4 Equilibrium Location Configurations

Our analysis in the previous section is summarized in the payo$\hat{\text{a}}$ matrix of Table 1, which allows us to solve the first-stage location game.

\[
\text{(Table 1 near here)}
\]

Since we have assumed that \( \frac{1}{2} \theta_1 > 0 \) it follows naturally that the location configurations \( fa;0 \) \( fb;0 \) \( ag; f0 \) \( bg \) cannot be SPE configurations. From Lemma 2 we also know that \( fa;ag \) and \( fb;bg \) cannot be SPE. If the two rms choose to locate in a single market they will choose different markets. Now consider the location configurations \( fa;bg \) and \( fb;ag \).

**Lemma 3** With uniform pricing the location configurations \( fa;bg \) and \( fb;ag \) are SPE if and only if

\[
\hat{\theta} > \hat{A}_1(\pm) = \frac{1}{2} \theta_m(\pm) + \frac{1}{2} \mu_m(\pm)
\] (18)

In (18) the term \( \frac{1}{2} \theta_m(\pm) \) is the loss of profit that the multimarket rm incurs in its captive market in order to enter its rival's market, while \( \frac{1}{2} \mu_m(\pm) \) \( \hat{\theta} \) is the additional profit that it earns from sales in the second market. Equation (18) then defines a lower limit on \( \hat{\theta} \), for each value of \( \pm \) above which neither rm will choose to enter the rival's market. It is clear that \( \hat{A}_1(0) = \frac{1}{2} \theta_0 \) and that \( \hat{A}_1(\pm) = \hat{\theta} < 0 \). Moreover, since \( P_{1b}^{fa, ab, ag} \) = \( c \) we know that \( \hat{A}_1(0) < 0 \). As a result, (18) implicitly defines an upper limit on \( \pm \) below which \( fa;bg \) and \( fb;bg \) cannot be SPE if set-up costs are \( \hat{\theta} = 0 \).

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6 Proofs of this and subsequent lemmas follow immediately from Table 1.
Now consider the location configurations \( f_a b; 0 g \) and \( f 0; a b g \). For these to be SPE requires \( \frac{1}{2} \Delta u (\pm) \mid A < 0 \) and \( \frac{1}{2} \Delta (\pm) \mid A < 0 \), again implicitly defining lower limits on \( A \). We know from Lemma 2 that \( \frac{1}{2} \Delta (\pm) \mid A < 0 \) is the binding constraint.

Lemma 4 With uniform pricing the location configurations \( f_a b; 0 g \) and \( f 0; a b g \) are SPE if and only if
\[
\dot{A} > \dot{A}_2 (\pm) = \frac{1}{2} \Delta u (\pm)
\] (19)

It follows from Lemma 2 that \( \dot{A}_2 (\pm) > \dot{A}_3 (\pm) \). Moreover, \( \dot{A}_2 (0) = \frac{1}{2} \Delta^0 \), \( \frac{1}{2} \Delta^0 (\pm) = \frac{1}{2} \Delta (\pm) < 0 \) and \( \dot{A}_2 (1) = 0 \). In other words, for \( \dot{A} > \dot{A}_2 (\pm) \) we have that \( f_a b; 0 g; f b; a g; f a b; 0 g \) and \( f 0; a b g \) are all SPE. By contrast, in the parameter region \( \dot{A}_2 (\pm) ; \dot{A}_1 (\pm) \) we have that \( f_a b; 0 g \) and its permutations are SPE while \( f a b 0 g \) and \( f 0; a b g \) are not. With uniform pricing it is more difficult to maintain a multimarket monopoly than for two firms to be single market monopolists, despite the fact that the resulting prices are identical so far as consumers are concerned. This is perhaps best interpreted as an example of Judd’s (1985) analysis. The multimarket rm is in a weaker position in being able to maintain its monopoly than is a single market monopolist because of the negative externality that uniform pricing imposes on the multimarket rm in confronting a rival in one of its markets.

For the location configuration \( f_a b; a b g \) to be SPE requires that \( \frac{1}{2} \Delta (\pm) \mid A > 0 \) and \( 2 \frac{1}{2} \Delta (\pm) \mid 2 \dot{A} > \frac{1}{2} \Delta u (\pm) \mid A \), each of which defines an upper limit on \( A \). From Lemma 2 the latter constraint is the binding constraint.

Lemma 5 With uniform pricing the location configuration \( f_a b; a b g \) is SPE if and only if
\[
\dot{A} < \dot{A}_3 (\pm) = 2 \frac{1}{2} \Delta (\pm) \mid \frac{1}{2} \Delta u (\pm)
\] (20)

It is easy to see that \( \dot{A}_3 (0) = \frac{1}{2} \Delta^0 \) and that \( \dot{A}_3 (1) = 0 \) so that there is a non-empty parameter range for which \( f_a b; a b g \) is SPE. While it is not possible to sign \( \frac{1}{2} \Delta u (\pm) = @\Delta (\pm) \) unambiguously, it is certainly the case that \( \dot{A}_2 (\pm) > \dot{A}_3 (\pm) \) as we should expect. Now consider the relative magnitudes of \( \dot{A}_3 (\pm) \) and \( \dot{A}_1 (\pm) \). For us to have \( \dot{A}_3 (\pm) > \dot{A}_1 (\pm) \) requires
\[
\frac{1}{2} \Delta^0 \mid \frac{1}{2} \Delta^0 (\pm) > \frac{1}{2} \Delta u (\pm) + \frac{1}{2} \Delta (\pm) \mid 2 \frac{1}{2} \Delta (\pm)
\] (21)
The right-hand-side of (21) tends to zero as $\pm 1$ whereas the left-hand side tends to the monopoly profit. As a result, there is a value of $\pm$ above which $fa; bg$, $fb; ag$ and $fab; abg$ are all SPE for some value of $\Delta$.

Finally, consider the asymmetric location configurations $fab; ag$, $fab; bg$, $fa; abg$ and $fb; abg$. By exactly the same arguments as those above, we have:

Lemma 6 With uniform pricing the location configurations $fab; ag$ and its permutations are SPE if and only if

\[
\Delta > \Delta_3 (\pm) \quad (22)
\]

and

\[
\Delta < \Delta_1 (\pm) \quad (23)
\]

Note that (22) and (23) are just the reverse of (20) and (18) respectively. For these to be consistent, therefore, requires that $\Delta_3 (\pm) < \Delta_1 (\pm)$ which is the reverse of (21). In other words, we should expect to find a value of $\pm$ below which these asymmetric locations configurations are SPE for some combinations of $\pm$ and $\Delta$.

The analysis of this section is illustrated in Figure 1. Multimarket operation has the potential to increase sales revenue but increases the intensity of price competition and incurs additional set-up costs. As a result, and as we might expect, for a given level of set-up costs $\Delta$, there is likely to be more multimarket contact when the degree of product differentiation is high since this softens price competition while maintaining sales. By contrast, when set-up costs are high or the degree of product differentiation is low we should expect to see much less multimarket contact. In the retail, telecommunications and service sectors to which our analysis applies, we would expect to see more multimarket contact either when set-up costs are relatively low or in cases where the competing firms offer products or services with distinctive characteristics.

(Figure 1 near here)

Note finally that there are prisoners' dilemma aspects to the equilibrium location configurations. Aggregate profit with the configuration $fab; abg$ is $4\frac{1}{2}d (\pm) + 4A$ while with $fa; bg$ aggregate profit is $2\frac{1}{2}d + 2A$. The former is less than the latter if $\Delta > \Delta_4 (\pm) = 2\frac{1}{2}d (\pm) + \frac{1}{2}A$. Since
$\hat{A}_1 (\pm) < \hat{A}_2 (\pm)$ it follows that in the region $\hat{A}_3 (\pm) < \hat{A}_4 (\pm)$ the SPE location configuration $fab, abg$ is Pareto dominated for the two firms by the configuration $fa; bg$.

4 Price Discrimination

Now assume that the firms are able to charge different prices in the markets in which they are active and are unable to commit to uniform pricing.\(^7\) This leads to a change in only one of the sets of location configurations considered in the previous section: the asymmetric configurations $fab, fbg, fa; abg$ and $fb, abg$.\(^8\) Take the location configuration $fab; bg$. Now firm 1 solves

$$\max_{p_1a, p_1b} \left( p_1a - c \right) q(p_1a, p_2 + \pm) + \left( p_1b - c \right) q(p_1b) \ i \ 2\hat{A} \quad (24)$$

while firm 2 solves

$$\max_{p_2} \left( p_2 - c \right) q(p_2; p_1 - \pm) \ i \ \hat{A} \quad (25)$$

The result is that firm 1 charges the unconstrained monopoly price $p^m$ in market $b$, earning the unconstrained monopoly profit $\frac{1}{2}\hat{A}$, while both firms charge the unconstrained Bertrand duopoly price $p^d$ in market $a$, earning profit $\frac{1}{2}\hat{A}(\pm)$, while both firms charge the unconstrained Bertrand duopoly price $p^d$ in market $a$, earning profit $\frac{1}{2}\hat{A}(\pm)$, while both firms charge $p^d$. Price discrimination removes a constraint on the multimarket firm in the price that it charges in its non-captive market. As a result, there is no longer a negative profit externality owing from the non-captive market to the captive market from any reduction in price in the non-captive market. This makes the multimarket firm a tougher competitor in the non-captive market, leading to lower prices for both firms.

Price discrimination, therefore, changes the payoff matrix to that given in Table 2 and changes the SPE location configurations.

Lemma 7 With price discrimination the location configurations $fa; bg, fb; ag, fab; 0g$ and $f0; abg$ are SPE if and only if

$$\hat{A} > \hat{A}_5 (\pm) = \frac{1}{2}\hat{A} (\pm) \quad (26)$$

and the location configuration $fab, abg$ is SPE otherwise.

\(^7\) We leave to subsequent analysis the question of whether firms would endogenously choose uniform or discriminatory pricing: see Thisse and Vives (1988).

\(^8\) In the location configuration $fab, abg$ symmetry leads to uniform pricing across the two markets even where price discrimination is feasible.
Clearly, $\hat{A}_5(\pm) = \frac{1}{4}\mu$, $\hat{A}_5(\pm) = 0$ and $\hat{A}_5(1) = 0$. First, note that price discrimination results in there being no parameter ranges in which $f_{a;bg}$ and its permutations are SPE while $f_{ab;0g}$ or $f_{0;abg}$ is not. Second, the asymmetric location configurations are no longer SPE. As we noted above, these configurations are supported with uniform pricing by the profit externality that such pricing imposes on the multimarket rm. This externality creates an incentive for a rival rm to enter only one of its multimarket rival’s markets rather than both. Removing the uniform pricing constraint removes the externality. While this, as we have noted, makes the multimarket rm a tougher competitor, it also removes the incentive for the rival rm to limit the markets that it enters. If it is profitable for the rival rm to enter one of the multimarket rm’s markets, it is profitable to enter both of these markets.

It is clear from Lemma 2 that $\hat{A}_3(\pm) < \hat{A}_5(\pm) < \hat{A}_2(\pm)$. In order for $\hat{A}_5(\pm) > \hat{A}_1(\pm)$ it is necessary that $\frac{1}{4}\mu_i > \frac{1}{4}\mu_i(\pm)$ and $\frac{1}{4}\mu_i(\pm) > \frac{1}{4}\mu_i(\pm)$. In other words, for the multimarket rm the negative profit externality in the captive market from uniform pricing must be greater than any positive profit impact it might receive from weaker price competition in the non-captive market. This is certainly true for $\pm$ “large enough” and is a weaker condition than (21). It follows that there is a wider parameter range for which $\hat{A}_5(\pm) > \hat{A}_1(\pm)$ than for which $\hat{A}_3(\pm) > \hat{A}_1(\pm)$. Indeed, it is likely that $\hat{A}_5(\pm) > \hat{A}_1(\pm)$ for all values of $\pm$. Figure 2 illustrates the equilibrium with price discrimination.

(Figure 2 near here)

The obvious question is whether price discrimination is better for consumers than uniform pricing. We have noted that $\hat{A}_5(\pm) > \hat{A}_3(\pm)$. Recall that the latter defines the upper bound on $\hat{A}$ below which $f_{ab;abg}$ is SPE when rns employ uniform pricing whereas the latter defines the upper bound on $\hat{A}$ below which $f_{ab;abg}$ is SPE when rns price discriminate. As a result, there is a non-empty parameter range $\hat{A}_5(\pm) < \hat{A}_3(\pm)$ (the shaded region in Figure 2) in which the SPE location configuration has more multimarket contact with price discrimination than with uniform
pricing, benefitting consumers through lower prices.

There is an equally unambiguous impact on firms. In the regions where consumers gain from price discrimination aggregate profit with price discrimination is \( 4^{\alpha'} (\pm) \cdot 4 \bar{A} \) whereas with uniform pricing it is either \( \frac{3}{2} Q^u (\pm) \cdot 3 \bar{A} \) or \( 2 \frac{1}{2} Q^u (\pm) - 2 \bar{A} \). Applying the constraints that determine the boundaries of the relevant regions it follows immediately that in the cases where price discrimination changes the SPE location configuration to \( \bar{f} \), \( \bar{a} ; \bar{b} \) firms lose from the additional competition that price discrimination induces. We can summarize this as:

**Theorem 1** Price discrimination results in more multimarket contact than does uniform pricing. The increase in multimarket contact benefits consumers but harms firms.

The intuition has been presented above. Price discrimination in oligopolistic markets of the type analyzed in this paper makes markets more competitive, benefitting consumers but harming firms.

Finally, note that we have a prisoners' dilemma in location choice for \( \bar{A} \), \( \bar{A} (\pm) ; \bar{A} (\pm) \).

## 5 An Example: linear demand

Suppose that the firms produce differentiated products for which inverse demand is linear, given by:

\[
p_{ik} = \theta_i (1 - \bar{q}_{ij} + \pm q_{jk}) i; j = 1; 2; k = a; b
\]  
(27)

where \( a; b > 0; \pm 2 [0; 1] \). Unit production costs are \( c \) per unit and a firm incurs set-up costs of \( F \) to open in either market. Inverting this system gives the direct demand functions:

\[
q_k = \frac{\gamma (1 \pm \frac{1}{2} \bar{p}_{ij} + \pm \bar{p}_{jk})}{(1 \pm \frac{1}{2})} i; j = 1; 2; k = a; b
\]  
(28)

As in the analysis above, we assume that consumers buy only in their local markets and that firms cannot export from one market to the other.

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9 For similar results in different contexts see, for example, Norman and Thisse (1996, 2000).

10 This is a commonly employed demand system. See, for example, Singh and Vives (1984), De Fraja and Norman (1993), Baldwin and Ottaviano (2001).
Standard analysis\textsuperscript{11} gives the equilibrium prices:

\[
p^m = \frac{\bar{\pi} + c}{2} , \quad p^d(\pm) = \frac{\bar{\pi}(1 \pm \delta) + c}{2} ,
\]

\[
p^a_{1a} = \frac{(\bar{\pi} + c)(1 \pm \delta)(4 + 3 \pm + c(2 + \pm)}{8} + \frac{c\pm}{8} + \frac{\pm}{2} \frac{\pm}{2} ,
\]

\[
p^a_{2a} = \frac{(\bar{\pi} + c)(1 \pm \delta)(4 + 2 \pm i + 4 + \pm 2)}{8} + \frac{\pm}{5} + \frac{\pm}{2} \frac{\pm}{2} \frac{\pm}{2} .
\]

At first sight this looks intractable, since there are five parameters. However, with this formulation, profits gross of set-up costs \( F \) are homogeneous of degree one in \( (\bar{\pi} \pm c)^2 \). As a result, no generality is lost if we make the normalization \( \bar{A} = F = (\bar{\pi} \pm c)^2 \) and let \( \bar{\pi} = (\bar{\pi} \pm c)^2 = 1 \). This gives the profits:

\[
\frac{1}{\gamma} \left( \frac{1}{\bar{A}} \right) = \frac{1}{4} \frac{1}{\bar{A}} \frac{1}{\bar{A}} \left( \frac{1}{\bar{A}} \right) = \frac{1}{4} \frac{1}{\bar{A}} \frac{1}{\bar{A}} \left( \frac{1}{\bar{A}} \right) ,
\]

\[
\frac{1}{\gamma} \left( \frac{1}{\bar{A}} \right) = \frac{1}{8} \frac{1}{\bar{A}} \frac{1}{\bar{A}} \left( \frac{1}{\bar{A}} \right) ,
\]

\[
\frac{1}{\gamma} \left( \frac{1}{\bar{A}} \right) = \frac{1}{8} \frac{1}{\bar{A}} \frac{1}{\bar{A}} \left( \frac{1}{\bar{A}} \right) ,
\]

\[
\frac{1}{\gamma} \left( \frac{1}{\bar{A}} \right) = \frac{1}{8} \frac{1}{\bar{A}} \frac{1}{\bar{A}} \left( \frac{1}{\bar{A}} \right) .
\]

In identifying the SPE location configurations it is clear that all the comparisons can be made in terms of the two parameters \( \pm 2 [0; 1] \) and \( \bar{A} 2 [0; 1\pm4] \). In this case it is always the case that \( \frac{1}{\gamma} \left( \frac{1}{\bar{A}} \right) < \frac{1}{\gamma} \left( \frac{1}{\bar{A}} \right) \).

Figure 3 illustrates the subgame perfect equilibrium location configurations with uniform pricing and Figure 4 illustrates the equilibria with discriminatory pricing. The shaded area in Figure 4 is the parameter region in which discriminatory pricing leads to an increase in multimarket contact. In this region it is easy to con..rm, as Theorem 1 indicates, that consumers gain from lower prices but rms lose from lower pro.ts.

\textsuperscript{11} Details can be obtained from the author on request.
6 Extensions

A number of extensions of the analysis are worth considering. We concentrate on three of these.

First, we could relax the assumption that the two markets are of equal size. The simplest way to do so would be to write the demand functions (1) as

$$q_{ik} = n_k q_i p_{ik}^l p_{jk}^l; i = 1, 2; j = a, b$$

where $$n_k$$ is the number of consumers in market $$k$$. The effect of doing so is easy to see. Suppose that $$n_a > n_b$$. Then, for a sufficiently large difference in market sizes we should expect to find that $$f_a; a$$ is preferred to $$f_a; b$$ as an equilibrium location configuration. It might also be possible that the profit externality in the asymmetric configuration $$f_{ab}; a$$ or $$f_a; ab$$ is sufficiently strong that these asymmetric configurations are not SPE for any parameter values. Our basic comparative statics would be largely unaltered, however, and it would remain the case that there is a parameter region in which a switch from uniform to discriminatory pricing would still benefit consumers and harm firms.

A second and more fundamental extension is to relax the assumption of there being no consumer arbitrage across the two markets. In other words, we can consider a situation in which a consumer in market a can buy a product in market b by incurring an additional cost $$t$$ per unit. In geographic space $$t$$ is just a transport cost. In a more general product or time-based space $$t$$ is the value of the utility loss that a consumer incurs in having to travel or eat at the “wrong” time or purchase a less than ideal product.

A full discussion of this case requires much more extensive analysis than can be justified here. We would have to consider, in particular, the relationship between the different location configurations and the critical values of $$t$$ below which consumer arbitrage is feasible. As a result, we focus on the case in which $$t$$ is sufficiently low that consumer arbitrage is feasible in every location configuration and in which demand is linear, given by (27).

It is simple to show that if we define $$\zeta = t/a$$ then all of the analysis can be conducted in
We can further show that consumer arbitrage is feasible in every location configuration provided that \( \zeta < \frac{2}{3} \). The profits in the various location configurations with no price discrimination are then:

\[
\begin{align*}
\gamma_{a;0g} &= \frac{(2i) (\zeta)^2}{8} i 2\zeta; \\
\gamma_{a;ag} &= \gamma_{a;bg} = \frac{2(2i) (\zeta i \pm \zeta)^2}{2i} i 2\zeta; \\
\gamma_{ab;ag} &= \frac{i 2(2i) (\pm i \zeta + \pm \zeta i \pm \zeta)^2}{2i} i 2\zeta; \\
\gamma_{ab;bg} &= \frac{2(2i) (\pm i \zeta + \pm \zeta)^2}{2i} i 2\zeta; \\
\gamma_{ab;ab} &= \frac{2(2i) (\pm i \zeta + \pm \zeta)^2}{2i} (1 + \zeta) i 2\zeta.
\end{align*}
\]

We illustrate the resulting equilibrium location configurations in Figure 5 for two cases: \( \zeta = 0.25 \) and \( \zeta = 0.5 \).

The qualitative impact of consumer arbitrage is not dramatic, but we can identify four main changes to the equilibrium location configurations. First, the firms are now indifferent between the location configurations \( f_{a; a} \) and \( f_{a; b} \), a result that is undoubtedly attributable to the linear specification. Second, there is a parameter range for which \( f_{a; 0g} \) is SPE. In other words, it is possible for one firm to monopolize the market by supplying consumers in both markets from a single “location”. Third, there is a smaller parameter range in which full multimarket contact, the configuration \( f_{ab; ab} \), is an equilibrium. The ability of consumers to arbitrage across markets reduces the incentive of the firms to incur the set-up costs necessary to have a direct presence in both markets. Fourth, price discrimination has a much less dramatic effect on the equilibrium: illustrated by the dotted line in Figure 4. Simply put, the ability of consumers to buy in both markets significantly reduces the profit externality of uniform pricing.

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12 Rowthorn (1992) also employs this “demand adjusted” measure of transport costs.
The third and last extension we consider is to return to the issue of tacit cooperation between the firms. The current literature considers how multimarket contact can sustain tacitly cooperative prices. In our analysis this is equivalent to firms choosing the location configuration \( f_a b; a b \) rather than \( f a; b \). We know from our analysis in sections 3 and 4 that this reduces static profit for both firms. As a result, such a switch in location configuration can be justified only if it facilitates price coordination. Our analysis then suggests where we are most likely to find such coordination: in sectors characterized by relatively high set-up costs and/or low degrees of product differentiation.

Rather than repeating the previous work on price coordination, we adopt a rather different approach in that we consider whether the firms can sustain tacitly cooperative locations. Specifically, we ask whether the firms might be able to sustain a location configuration in which neither firm invades the other firm's market.

Assume that there is no consumer arbitrage between markets. Our analysis in sections 3 and 4 shows that we have a prisoners' dilemma game in location choice for \( \bar{A}_2 [\bar{A}_4 (\pm) ; \bar{A}_5 (\pm)] \) with no price discrimination and \( \bar{A}_2 [\bar{A}_4 (\pm) ; \bar{A}_5 (\pm)] \) with price discrimination. Now suppose that the firms are involved in an infinitely repeated game, with the analysis of sections 3 and 4 characterizing competition in each period. Consider the following trigger strategy for firm 1. “I shall continue with location strategy \( a \) in the current period so long as you have maintained location strategy \( b \) in all previous periods. However, if you have ever switched to strategy \( a b \) I shall switch to \( a b \) forever.”

A particular advantage of this strategy is that it is triggered by a deviation that is easy to detect. In addition, this type of cooperation may be easier to hide from the regulatory authorities than agreements on prices. Standard analysis shows that the suggested trigger strategy is a subgame perfect equilibrium strategy sustaining the location configuration \( f a; b \) provided the discount

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13 This implies, of course, that \( \bar{A} \) be treated as per-period fixed costs of establishing a production facility.
factor \( R \) satisfies:

\[
R > R^u = \frac{\frac{\mu_i^u}{\mu} \left( \mu^u \right)^i \lambda^u}{\frac{\mu_i^d}{\mu} \left( \mu^d \right)^i \lambda^d} \tag{29}
\]

with uniform pricing and

\[
R > R^d = \frac{\frac{\mu_i^d}{\mu} \left( \mu^d \right)^i \lambda^d}{\frac{\mu_i^u}{\mu} \left( \mu^u \right)^i \lambda^u} \tag{30}
\]

with discriminatory pricing. Note that \( R^u < 0 \) as \( \lambda^u > \lambda^d \), \( R^d > 0 \) as \( \lambda^d > \lambda^u \), \( R^u > R^d \) as \( \lambda^u > \lambda^d \). In other words, there is a non-empty parameter range in which mutual forbearance in location choices is sustainable as a subgame perfect equilibrium for the repeated location game. Moreover, if \( \lambda^u > \lambda^d \) as we would generally expect, price discrimination makes it more likely that the firms will be able to maintain their cooperation in location choice. In other words, price discrimination could be a mixed blessing. On the one hand it increases the possibility that non-cooperative firms will choose multimarket contact, benefitting consumers. On the other hand, it also makes it more likely that these firms might be able to sustain an agreement not to invade each other's markets.

7 Concluding Remarks

There are many situations where, if firms wish to supply consumers in particular markets, they must establish direct operations in those markets. Retail, banking, telecommunications and restaurant services are but a few of the many examples of such non-tradeable goods. A firm's choice of time slots in which it operates is another, provided that consumers do not arbitrage across time. We have shown that the degree of multimarket contact that these firms choose is determined by the degree to which their products are differentiated and the set-up costs necessary to establish operations. Multimarket contact is likely to be more extensive when the firms offer distinctive products and when set-up costs are not "too high".

We have further shown that uniform pricing typically leads to less multimarket contact than does price discrimination. With uniform pricing a firm that invades a rival's market suffers a negative profit externality that does not exist with price discrimination. As a result, price
competition is likely to be softer with uniform pricing than with discriminatory pricing. The direct result is that consumers gain when competing rms adopt discriminatory prices while the rms lose. Introducing costly consumer arbitrage across markets changes our results to some extent, in particular, reducing the possibility that rms will invades each other’s markets, but leaves our basic conclusions unaltered.

As might have been expected, there is a non-trivial parameter range in which the rms earn greater profits with local monopolies but in which they adopt full multimarket contact. This provides an alternative view of the relationship between multimarket contact and tacit collusion. The current literature suggests that rms seek such contact as a mechanism for sustaining tacitly cooperative prices. We have shown, by contrast, that multimarket contact can emerge naturally as a subgame perfect equilibrium. This does not preclude the possibility that rms will subsequently seek for means to cooperate in prices. However, our analysis suggests that the strategic use of location choice as a means to sustain price coordination is only likely to be found in markets with high set-up costs and a low degree of product differentiation.

Where these conditions do not apply, our analysis points to the possibility that rms will coordinate in location choice rather than prices. Specifically, we have shown that there are cases in which an agreement by rms not to invade each other’s markets is sustainable by a trigger strategy based upon an easily observable deviation. This implies that the degree of multimarket contact is a potentially ambiguous indicator of the existence of tacit (or explicit) coordination. Once again, there is no simple market test available to the regulatory authorities in their attempts to undermine cooperation between rms.
References


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Table 2: Payoff Matrix - Price Discrimination

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Figure 1: Location Equilibrium: no price discrimination

Figure 2: Location Equilibrium: price discrimination
Figure 3: Linear Demand: no price discrimination

Figure 4: Linear Demand: price discrimination
Figure 5: Location Equilibrium with Consumer Arbitrage