Heterogeneity, adverse selection and valuation with endogenous labor supply☆

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Abstract

This paper considers models of intratemporal consumption–labor choice and intertemporal consumption choice under heterogeneity and private information in preferences towards labor. We show that market regime regarding unemployment insurance is important to determine the effects of heterogeneity and private information on allocations and valuations. There are two main results. First, intertemporal choice can mitigate adverse selection. Second, in countries where unemployment insurance is generous capital markets should have low usage and the risk-free rate of return is low. However, in countries where unemployment insurance is less generous, capital markets should have more usage and the risk-free rate of return is higher.

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1. Introduction

In this paper, we consider a dynamic two-period model with endogenous labor supply and potential for adverse selection based on private information of individual preferences. We use the model to draw conclusions about consumption and labor valuation and capital market activity.

Single period models of market activity with heterogeneity in preferences regarding labor supply show that under private information an important issue of adverse selection arises: Individuals have an incentive to misrepresent, e.g. Prescott and Townsend (1984), Townsend (1987) and more recently Bianconi (2001). We introduce intertemporal choice adding a dimension of consumption smoothing to the intratemporal problem of consumption–work choice. We show that the introduction of intertemporal choice can have important effects on efficient allocations and valuations. In our basic framework, there are several units, regions, or countries and potential differences within and across units.

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Individuals may use capital markets for consumption smoothing and there are potential differences in the preference for labor supply. We allow trade in lotteries as a revelation mechanism to avoid adverse selection, and consider allocations with tradable labor income.¹ We present two main results. First, the introduction of intertemporal choice can mitigate and/or eliminate adverse selection under private information. Second, the valuation of consumption and labor and the use of capital markets can vary substantially across different degrees of heterogeneity and different regimes of unemployment insurance.

This paper is a dynamic extension of Bianconi (2001), who finds that unemployment insurance regimes matter for asset allocation in a static framework. In the dynamic case, unemployment insurance matters along with the consumption smoothing problem. Individuals start with neither borrowing nor lending but use their ability to borrow/lend to smooth consumption. In the static case under private information, trade in lotteries (contracts) for employment can mitigate adverse selection providing a classic separating mechanism. In the dynamic case, we show that the introduction of trade in another good at another date (intertemporal choice) can alone mitigate the uneven distribution of consumption and work, which leads to adverse selection under private information. We further examine trade in lotteries (contracts) for employment in the dynamic case when the parameter space allows adverse selection to occur.

In our models, the regime regarding unemployment insurance plays an important role in determining efficient allocations and valuations.² The main result is that a regime of full unemployment insurance implies that individuals use capital markets only moderately. However, without unemployment insurance capital markets are used more heavily because individuals who eventually do not work must save to make up for the loss in future output. An earlier paper by Diamond and Mirrlees (1978) discusses the desirability of private saving in the presence of social insurance mechanisms. Here we find that, for sufficiently high elasticity of intertemporal substitution (high demand for consumption smoothing), private saving due to lack of full unemployment insurance (incomplete markets) is welfare improving over full unemployment insurance. However, for other parameter configurations this may not be the case. While authors such as Mulligan (2001) find negligible quantitative aggregate implications of indivisibilities in labor supply, we show that intertemporal trade and private information together with alternative regimes for unemployment insurance can have rather significant effects on allocations and valuations across units, regions or countries.

In particular, we first show that a necessary and sufficient condition for an uneven distribution of consumption and work across units to occur, under full information, is that the type with higher elasticity of intratemporal substitution between consumption and work (or less risk averse in labor supply) must be a borrower in the first period. This provides an appropriate parameter space for the study of economies with private information where adverse selection occurs. Then, we examine schemes of trade in lotteries (contracts) for employment under alternative unemployment insurance arrangements. Our analysis clearly shows that in countries where unemployment insurance is generous, capital markets have low use and the risk-free rate of return is low. However, in countries where unemployment insurance is less generous, capital markets have more usage and the risk-free rate of return is higher. This is an important hypothesis that requires further empirical testing.

The paper is organized as follows. Section 2 considers the basic structure while Section 3 gives the preliminaries on efficient allocations under full information with homogeneous types. Section 4 examines the alternative models with asymmetric information and alternative markets structures regarding unemployment insurance. Section 5 examines the numerical simulation of the alternative economies, and Section 6 examines the relationship between the allocations and the model equilibrium. Section 6 concludes.

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¹ Prescott and Townsend (1984) and Townsend (1987) present the revelation mechanism used in this paper. Bianconi (2001) is a recent application. Another recent related model is Mulligan (2001) for the case of indivisibilities in labor supply, not adverse selection under private information. Other recent related papers are Gomes, Greenwood, and Rebelo (2001) and Alvarez and Veracierto (1999). The paper by Chan and Viceira (2000) presents a model where labor income is not tradable, and Bodie, Merton, and Samuelson (1992) is an early attempt to study the potential effects of labor choice on asset allocation.

² The notion of unemployment insurance discussed in this paper refers to the case where the lottery for employment comes with a clause of either full consumption regardless the outcome of the lottery, or no consumption if the outcome is unemployment. Thus in this paper, we examine the polar cases of either 100% replacement (full unemployment insurance) or no replacement (no unemployment insurance), e.g. Martin (1996). Other notions of unemployment insurance consider that a productivity shock occurs with some probability that the individual may loose her employment and so wishes to purchase unemployment insurance against that “bad” outcome. The papers by Gomes et al. (2001) and Alvarez and Veracierto (1999) consider this problem of unemployment insurance with alternative replacement ratios. This paper does not consider the productivity shock channel; but instead the heterogeneity in individual preferences channel.
2. Basic structure

The model economies of this paper comprise one good and are cast in a dynamic two-period framework with a finite number of units indexed by \( j \in J : j = 1, 2, \ldots, J \). Each unit may be referred as an island, a region or a country inhabited by a large (countably infinite) number of identical individuals. A priori there may be no differences within units, but there are potential differences across units. In the first period, the units have identical endowments, \( y > 0 \), each consume \( c_{1j} > 0 \), and spend the entire time at leisure. Each unit starts with no borrowing and lending, but capital markets are open for trade in standard one-period contracts (bonds). In the second period, the unit consumes \( c_{2j} > 0 \) and spends a fraction of the time at work, \( n_j \geq 0 \).

A typical unit has a production technology for second period output given by

\[
y_j = z_j f(n_j)
\]

where \( y_j \) is the output, per unit of capital, produced by unit \( j \), \( f \) is a strictly increasing and strictly concave function identical for all \( j \) (\( f' > 0, f'' < 0 \)), and \( z_j \) is the total productivity level of the technology. The differences in productivity across units may be potentially unobservable, however we assume the existence of an organized asset market that reveals the market-relevant information of the unit’s productivity as in the recent contribution of Berliant and De (1998). In what follows, we assume that \( f(n_j) \) takes the specific form

\[
f(n_j) = (n_j)^\alpha
\]

for \( \alpha \in (0, 1) \). Capital markets are perfectly integrated across units, labor is assumed immobile but labor income is assumed to be tradable, see e.g. Leung (1995), and Chan and Viceira (2000).

The typical unit utility function is given by the function

\[
\Omega^j(c_{1j}, c_{2j}, n_j) = u(c_{1j}) + \beta[u(c_{2j}) + v^j(n_j)]
\]

where \( \beta \in [0, 1] \) is the subjective discount factor common to all units. The function \( u \) is assumed to be strictly increasing and strictly concave (\( u' > 0, u'' < 0 \)) and identical across individuals. The function \( v \) is assumed to be strictly decreasing and concave (\( v''' > 0, v'''' \leq 0 \)). The intratemporal elasticity of substitution between consumption and leisure may differ across individuals depending upon the sign of the differential \( v'''' \leq 0 \). The main assumption in (3a) is that consumption and labor are separable in second period utility; in effect as in Mulligan (2001) we assume complete separation between \( c \) and \( n \) in utility. As mentioned, the source of private information in this model will be regarding the sign of the second differential \( v'''' \leq 0 \), i.e. the concavity of the utility function with respect to labor supply in the second period. Throughout the analysis, we assume preferences of the special form

\[
\Omega^j(c_{1j}, c_{2j}, n_j) = (1/1 - \sigma) c_{1j}^{1-\sigma} + \beta[(1/1 - \sigma)c_{2j}^{1-\sigma} - \delta(n_j^{1+\gamma_j})/(1 - \gamma_j)], \quad \text{each } j
\]

where \( \sigma \in (0, \infty) \) gives the (common across all, \( \sigma = 1 \) indicates logarithmic utility) coefficient of relative risk aversion with respect to consumption (or the inverse of the elasticity of intertemporal substitution in consumption across periods), \( \delta > 0 \) is a constant reflecting proportional disutility of labor, and \( 0 \leq \gamma_j \) gives the curvature of the utility function relative to labor supply. For this utility function, the elasticity of intratemporal substitution between consumption and leisure \((1-n^2)\) in the second period is given by the formula

\[
v^j(c_{2j}, n_j) = -(c_{2j}^{\sigma-1} \delta n_j^{1+\gamma_j} - 1)/(c_{2j}^{\sigma-1} \delta n_j^{1+\gamma_j} + \gamma_j), \quad \text{each } j.
\]

which will be evaluated below.

3. Efficient allocations: preliminaries

In the absence of heterogeneity across units and with full information, there is no reason for trade to occur. The First and Second Fundamental theorems insure that a competitive equilibrium is Pareto Optimal and that a planner can
appropriately find prices and quantities that replicate the competitive equilibrium. The Pareto efficient allocation can be obtained by maximizing a social welfare function subject to period-by-period resources constraints, or

\[
\begin{align*}
\text{Max}_{\{c_{ij}, c_{2j}\}} \quad & \mathbb{E}[\sum \omega_j \Omega (c_{1j}, c_{2j}, n_j)] \\
\text{subject to} \quad & \sum \pi_j c_{1j} - y \leq 0 \quad \forall j \in J \quad \text{(4a)} \\
& \sum \pi_j (c_{2j} - z_j f(n_j)) \leq 0 \quad \forall j \in J \quad \text{(4b)}
\end{align*}
\]

where \(\omega_j\) are arbitrary welfare weights satisfying \(\{\omega_j: \omega_j \geq 0, j=1, 2, \ldots, J, \sum_j \omega_j = 1\}\), \(\pi_j\) are resource weights to account for possible differences in size of units satisfying \(\{\pi_j: \pi_j \geq 0, j=1, 2, \ldots, J, \sum_j \pi_j = 1\}\), and \(\mathbb{E}\) is the expectations operator over possible randomness in technology and preferences.

In this framework, efficiency requires

\[
q_2/q_1 = \mathbb{E}[u'(c_2)/u'(c_1)] \quad \forall j = 1, 2, \ldots, J \quad \text{(5a)}
\]

where \(\{q_1, q_2\} \geq 0\) are Lagrange multipliers attached to the resources constraints, i.e. the expected marginal rate of substitution in consumption across periods equals the marginal rate of transformation in the capital market expressed as the relative price of consumption across periods. Note from expression (5a) that the growth in consumption is identical across all \(j=1, 2, \ldots, J\), the usual full “risk sharing” characteristic of frictionless economies. Labor/leisure choice in the second period yields

\[
\mathbb{E}[v'(n_j) + (c_2) z_j f''(n_j)] = 0 \quad \forall j = 1, 2, \ldots, J \quad \text{(5b)}
\]

i.e. the marginal rate of substitution between consumption and work in utility is equal to the marginal rate of transformation in production for the second period. The two constraints (4a), (4b) hold with equality, and we obtain solutions for \(\{c_{1j}, c_{2j}, n_j, q_1, q_2\}\) as a function of technology and preference parameters. In this case, if \(\omega_j = \pi_j = 1/J\) all \(j\), \(z_j = z\) all \(j\), then \(c_{1j} = c_1, c_{2j} = c_2, n_j = n\) all \(j\), the perfectly pooled equilibrium: All market participants are identical and there is no reason to engage in trade across units, regions or countries.

Consider next the case of heterogeneity in preference towards labor across units, but full information, i.e. the heterogeneity is public information. Each unit has only one type and differences across units reflect differences across types. There are gains from trade in this case. The Pareto efficient allocation is obtained by maximizing the social welfare function subject to the resources constraints as in (4) yielding Pareto efficiency as in (5a)–(5b).

The central assumption describing heterogeneity regards the preference towards (dis)utility of labor. Let there be two units or types \(J=2\), with identical weights \(\omega_j = \pi_j\), identical productivity \(z_j = z\) all \(j\), and preference structure [recall (3b)]

\[
\begin{align*}
\bullet j = a: \quad & v^a(n_a) = 0 \Rightarrow \gamma_a = 0 \\
\bullet j = b: \quad & v^b(n_b) < 0 \Rightarrow \gamma_b > 0.
\end{align*}
\]

For type \(j=a\) preference is linear in labor supply and for type \(j=b\) it is strictly concave in labor supply. This difference in preferences implies that, ceteris paribus, for type \(j=a\), the elasticity of intratemporal substitution between consumption and leisure is larger than for type \(j=b\), e.g. (3c). In particular, the linearity in labor supply implies that \(j=a\) is ‘risk neutral’ in labor supply whereas the strict concavity for \(j=b\) implies ‘risk aversion’ in labor supply. The strict concavity with respect to consumption imply that both demand consumption smoothing, i.e. both are risk averse in consumption. In terms of labor supply, the more risk averse individual, \(v^b(n_b) < 0\), will prefer to work more hours instead of facing a gamble that would give average disutility; whereas the risk neutral type, \(v^a(n_a) = 0\), will be indifferent between the certain and the gamble. Hence, for the ‘risk neutral’ individual, the disutility of labor is proportional and the individual is more sensitive to changes in labor supply in terms of its utility costs. For the ‘risk
averse” individual, the disutility of labor is less than proportional and she is less sensitive to variation in labor supply in terms of its utility costs.\(^3\)

The introduction of heterogeneity has important implications for efficient allocations. To see this, first, consider a framework without the choice of first period consumption \((c_{1j})\) as in Bianconi (2001). In this special case, there is no possibility of intertemporal trade but, given the preference structure (6) individuals engage in intratemporal trade to negotiate differences in preference towards labor supply. The efficient allocation across units takes the form

\[
c_{1j} = y_j, \quad \text{all } j; \quad c_{2j} = c_2, \quad \text{all } j; \quad n_a < n_b.
\]

In this intratemporal allocation, both consume the same amount in the second period, but individual \(j = b\), which is risk averse in labor supply, works more.

The introduction of intertemporal trade, through the availability of another good at another date, has a nontrivial effect on the efficient allocation across units and has the potential to mitigate and/or eliminate the uneven allocation obtained in (7). We present below conditions under which the uneven allocation of expression (7) is preserved in the presence of both intratemporal and intertemporal trading opportunities.

**Proposition 1.** A sufficient and necessary condition for an allocation with \(n_a < n_b\) to occur, under technology structure (2), preference structure (6) and no sources of risk, is that \(c_{1a} > c_{1b}\) or \(c_{2a} > c_{2b}\), i.e. the individual less risk averse in labor supply, \(j = a\), must be a borrower in the first period.

**Proof.** The proof is simple. Capital market efficiency (5a) implies \(u'(c_{1a})/u'(c_{1b}) < u'(c_{2a})/u'(c_{2b})\). Hence,

\[
c_{1a} > c_{1b} \iff c_{2a} > c_{2b} \iff u'(c_{1a})/u'(c_{1b}) < 1 \iff c_{2a} > c_{2b} \iff u'(c_{2a})/u'(c_{2b}) < 1.
\]

From the labor/leisure margin (5b),

\[
v^a(n_a) + u'(c_{2a})f'(n_a) = v^b(n_b) + u'(c_{2b})f'(n_b) \iff v^a(n_a) - v^b(n_b) = z[u'(c_{2b})f'(n_b) - u'(c_{2a})f'(n_a)] < 0
\]

where the inequality follows from the preference structure (6), and thus follows

\[
u'(c_{2a})/u'(c_{2b}) > f'(n_b)/f'(n_a).
\]

From (8), \(c_{1a} > c_{1b}\) or \(c_{2a} > c_{2b}\) imply \(u'(c_{2a})/u'(c_{2b}) < 1\) which from (9) implies \(f'(n_b) < f'(n_a)\), or \(n_b > n_a\). When \(c_{1a} > c_{1b}\) or \(c_{2a} > c_{2b}\), resources constraints (4a), (4b) require individual \(j = a\) to be a borrower. \(\square\)

Ultimately, for Proposition 1 to be satisfied and for \(n_a < n_b\) to occur in the presence of both intratemporal and intertemporal trade, the parameter space must be such that the individual less risk averse in labor supply chooses to borrow in the first period. Under preference structure (6), the condition is that the risk neutral individual \(j = a\) must consume more than her endowment in the first period, and thus work less in the second period. The risk averse individual \(j = b\) must save in the first period and work more in the second period.

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\(^3\) It is useful to note that, in this model, the heterogeneity in disutility of labor is isomorphic to heterogeneity in the degree of diminishing marginal physical product in production. To see this, define the disutility of work as \(d_j = (n_j^{1+\gamma})/(1+\gamma)\) which gives labor supply in terms of the disutility of labor as

\[
n_j = (1 + \gamma_j)^{1/1+\gamma_j}/d_j^{1/1+\gamma_j}.
\]

Substituting labor supply into the production function yields

\[
y_j = (1 + \gamma_j)^{\gamma_j^{1/1+\gamma_j}}/d_j^{(1+\gamma_j)}
\]

and the model is isomorphic to individuals with welfare function

\[
\Omega(c_{1j}, c_{2j}, n_j) = (1/1-\sigma)c_{1j}^{1-\sigma} + \beta[(1/1-\sigma)c_{2j}^{1-\sigma} - \delta d_j]
\]

and different degrees of marginal physical products in production. In this alternative interpretation, the planner can observe output \(y_j\), but not “effort” \(d_j\). Since, \(d_j\) enters linearly in utility, heterogeneity in disutility of labor is isomorphic to heterogeneity in the degree of marginal productivity in production.
As an example, consider the following parameter configuration:
\[
\{\omega_j = \pi_j = 1/2, \alpha = 0.6, \beta = 0.95, \delta = 2.5, \gamma_a = 0, \gamma_b = 0.75, \sigma = 1.25, \gamma = 1, z = 2.534\}
\]
This parameter set yields an efficient allocation:
\[
\{c_{1a} = 1.456, c_{1b} = 0.544, c_{2a} = 1.977, c_{2b} = 0.739, n_a = 0.034, n_b = 0.902, y_a = 0.335, y_b = 2.381\}
\]
Individual \( j = a \) works and produces less, consumes more in both periods, and borrows in the first period \((c_{1a} = 1.456 > \gamma = 1)\), all relative to individual \( j = b \). We provide a brief sensitivity analysis by changing the parameter governing intertemporal substitution, \((1/\sigma)\) increases to 2 indicating lower elasticity of intertemporal substitution and lower demand for consumption smoothing for both individuals, all other parameters remain the same:
\[
\{\omega_j = \pi_j = 1/2, \alpha = 0.6, \beta = 0.95, \delta = 2.5, \gamma_a = 0, \gamma_b = 0.75, \sigma = 2, \gamma = 1, z = 2.534\}
\]
This alternative parameter set yields an efficient allocation:
\[
\{c_{1a} = 1.081, c_{1b} = 0.919, c_{2a} = 1.279, c_{2b} = 1.087, n_a = 0.084, n_b = 0.561, y_a = 0.574, y_b = 1.792\}
\]
The result relative to the case where \( \sigma = 1.25 \) is clear. The uneven distribution of work and consumption diminishes considerably, indicating that an increase in \( \sigma \) can mitigate the uneven distribution, i.e. a lower demand for consumption smoothing by both types can mitigate (and even reverse) the uneven distribution of consumption and work across individuals.

To sum, the presence of capital markets under heterogeneity in preferences towards labor supply can plausibly mitigate and/or eliminate the uneven distribution of consumption and work. Under a parameter space that satisfies Proposition 1, we obtain uneven distributions of consumption and work, but other sectors of the parameter space can easily eliminate it. In what follows, we assume that the parameter space is such that the uneven distribution initially occurs.

4. Heterogeneity, adverse selection, efficiency and market completeness

We now proceed by examining alternative cases relating to the information structure about potential differences in preferences, market regimes of unemployment insurance and the effects of open intertemporal capital markets on efficient allocations and valuations, under conditions satisfying Proposition 1.

4.1. Private information in individual preferences with heterogeneous types: full unemployment insurance

We introduce private information into the previous model under conditions satisfying Proposition 1. In this case, individual differences across units are private information of the specific unit and there may be differences in allocation ex-ante versus ex-post. A feasible and implementable allocation requires incentive compatibility constraints of the form
\[
\Omega^i(c_{1j}, c_{2j}, n_j) \geq \Omega^i(c_{1i}, c_{2i}, n_i) \quad \text{for all } j, i \in J, i \neq j
\]
\[(10)\]
i.e. individual of unit \( j \) when faced with alternative consumption–labor supply bundles in the second period will have an incentive to reveal her type truthfully, and she will have an incentive not to misrepresent her preferences towards labor. Under conditions that satisfy Proposition 1, with private information, the model in Section 3 is such that the incentive compatibility constraints will be violated ex-ante because type \( j = a \) consumes more and works less while type \( j = b \) works more and consumes less. Therefore, under private information, type \( j = b \) will have an incentive to misrepresent as type \( j = a \), and enjoy less work and more consumption. This is a classic adverse selection problem, e.g. Akerlof (1970). Technically, for utility \( u \) strictly concave in consumption, the consumption–labor supply possibility set is not convex, e.g. Prescott and Townsend (1984).4

4 The strict concavity of \( u \) is crucial for the non-convexity to arise. If \( u \) were to be linear, no differences in labor supply would occur and convexity would not be compromised. The lottery scheme presented below is based on Prescott and Townsend (1984), and other applications of lotteries may be found in Rogerson (1988), Besley, Coate, and Loury (1994), Bianconi (2001). For a comprehensive exposition of general equilibrium with lotteries, see Townsend (1987).
The revelation mechanism used here to avoid the adverse selection problem is to introduce a lottery scheme that convexifies the consumption–labor supply possibilities set.\(^5\) Denoting the consumption–labor supply bundles \((c, n) \in \mathcal{L}\) where \(\mathcal{L}\) is the consumption–labor possibility set, the analogous to the incentive compatibility constraints (10) with the introduction of the lottery scheme are

\[
\sum_{(c,n)\in\mathcal{L}} \phi_j(c,n) \Omega^i(c,n) \geq \sum_{(c,n)\in\mathcal{L}} \phi_i(c,n) \Omega^j(c,n) \quad \text{for all } j, i \in \mathcal{J}, i \neq j
\]

where \(\phi_j(c,n) \geq 0, \sum_j \phi_j(c,n) = 1\). Hence, \(\phi\) is the lottery for bundle the consumption–labor \((c, n)\). The incentive compatibility constraints in (11) are linear in the lottery and yield a convex consumption–labor possibility set. A consequence of introducing the revelation mechanism through the lottery scheme is that ex-ante and ex-post allocations may differ. Ex-ante, all individuals in all units are identical in expectations but ex-post the relevant differences are realized.

Consider again two units \(J = 2\), with \(\omega_j = \pi_j\), and preference structure as in (6). The revelation mechanism consists of introducing a lottery in the labor supply of individuals of unit \(j = a\) to make it unattractive to individuals of unit \(j = b\) who are risk averse, while not affecting the decisions of \(j = a\) who are risk neutral. Let the lottery for \(j = a\) be a contract with the firm with the following terms:

- with probability \((1 - \phi)\): \(n_a = 0\);
- with probability \(\phi\): \(n_a = n > 0\);
- \(\phi\) gives full unemployment insurance to the holder;

for \(\phi \in (0, 1)\), and \(n > 0\) given. The lottery ticket gives every holder full unemployment insurance, thus there are complete markets in unemployment insurance.\(^6\) The effective hours worked will be \(\phi \times n\) and every individual of unit \(j = a\) will receive ex-post a full wage \(z_a f'(\phi n)\) whether working or not. This assumes that markets provide full insurance at actuarially fair prices, e.g. Hansen (1985), Marcet, Obiols-Homs, and Weil (1998).

The expected utility for \(j = a\), ex-ante, is \(\Omega^a(c_{1a}, c_{2a}, \phi n)\) and \(j = a\) maximizes expected utility by choice of probability \(\phi\). Ex-ante allocations can be obtained as solutions to the social problem\(^7\)

\[
\max_{\{c_{1a}, c_{2a}, c_{1b}, c_{2b}, \phi, n\}} \{ \omega_a \Omega^a(c_{1a}, c_{2a}, \phi n) + \omega_b \Omega^b(c_{1b}, c_{2b}, nb) \}
\]

subject to

\[
\sum_j \pi_j c_{ij} - y \leq 0 \tag{12a}
\]

\[
\pi_a[c_{2a} - z_a f'(\phi n)] + \pi_b[c_{2b} - z_b f(n_b)] \leq 0 \tag{12b}
\]

where in the second period resources constraint, \(\phi\) enters as the proportion of individuals of unit \(j = a\) who actually work. Note that the social planner knows the location of individuals across units, but due to private information must give individuals the right incentive to reveal truthfully. Hence, the contract is offered to all across units, and gives the right incentive for all in each unit to reveal truthfully. In effect, there is no a priori intra-unit differences, and, ex-ante, no

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\(^5\) In areas of the parameter space where Proposition 1 is not satisfied, the lottery mechanisms studied in what follows would still be valid under the assumption of indivisibilities in the labor market as in the paper by Mulligan (2001); see e.g. Hornstein and Prescott (1989) for a review of indivisibilities and lotteries.

\(^6\) We do not consider any potential moral hazard problem relating to the work effort in the presence of full insurance here. The papers by Hansen and Imrohoroglu (1992) and Atkeson and Lucas (1995) present models where the moral hazard problem in unemployment insurance is fully analyzed.

\(^7\) The validity of the First and Second Fundamental Theorems for the economies with lotteries and private information used in this paper are studied in detail in Prescott and Townsend (1984), hence not discussed here.
inter-unit differences as well. However, ex-post both intra-unit and inter-unit differences will arise. A solution to (12) yields Pareto efficiency ex-ante for all units, or

\[ q_j/q_1 = E[\beta u'(c_{j1})/u'(c_{j2})] \quad \text{all } j = a, b \]  

(13a)

where, as before, \{q_1, q_2 \geq 0\} are the Lagrange multipliers attached to the resources constraints; and

\[ E[\Delta'(\phi n) + u'(c_{2a})z_{a}f'(\phi n)] = 0 \]  

(13b)

\[ E[\Delta'(n_b) + u'(c_{2b})z_{b}f'(n_{b})] = 0. \]  

(13c)

Indeed, this ex-ante allocation is identical to the full information allocation with heterogeneity in Section 3 when we set

\[ n_a = \phi n. \]  

(13d)

Hence, ex-ante Pareto efficiency holds and the lottery makes everyone better off in expectations. The ex-post allocation in this case is also efficient. The individuals in unit \( j = a \) are subdivided into the fraction \( (1 - \phi) \) who do not work, but due to the complete markets in unemployment insurance receive a full wage \( z_{a}f'(:n) \), and the fraction \( \phi \) that work \( n \) hours receiving the same wage \( z_{a}f'(:n) \).

Because of the linearity of the utility function of all \( j = a \) in \( \phi \) (risk neutrality), and separability between consumption and labor, we have that

\[ v^{a}(\phi n) = v^{a}(0) = \text{constant} \]  

(14)

i.e. independent of \( \phi n \) ex-post. Thus, efficiency ex-post holds for all \( j = a \), that is

\[ E[\Delta'(\phi n) + u'(c_{2a})z_{a}f'(\phi n)] = E[v^{a}(0) + u'(c_{2a})z_{a}f'(\phi n)] = 0. \]  

(15)

For all \( j = b \), efficiency holds as well, e.g. expression (5b). To sum, for \( v^{a}(n_a) = 0, v^{b}(n_b) \), separable utility between consumption and labor, and private information, ex-ante efficiency is consistent with ex-post efficiency with lotteries as a revelation mechanism.

However, differences in risk aversion towards labor in preferences matter for ex-post efficiency. Consider the model above with a slight modification in the preferences described in (6), within the parameter space that satisfies Proposition 1. Let \( v^{a}(n_a) = 0, v^{b}(n_b) \). Therefore, individuals in unit \( j = a \) are uniformly less risk averse than \( j = b \) or similarly have a uniformly higher elasticity of intratemporal substitution between consumption and leisure. The only difference from above is that now both types are risk averse. Individual preferences are private information and the revelation mechanism is identical: introduce a lottery for \( j = a \) to make it unattractive for \( j = b \), the more risk averse, while acceptable to \( j = a \), the less risk averse. The important issue here is the difference across units not the specific risk neutrality versus risk aversion per se.

For small risk aversion in labor supply of \( j = a \), ex-ante Pareto Efficiency holds in this case as well: for all of \( j = a \), ex-ante expression (13a) holds and for all of \( j = b \) ex-ante expression (13b) holds. However, ex-post allocations may not be the same. The individuals in unit \( j = a \) are subdivided into the fraction \( (1 - \phi) \) who do not work, but due to the complete markets in unemployment insurance receive a full wage \( z_{a}f'(:n) \), and the fraction \( \phi \) that work \( n \) hours and receive the same wage \( z_{a}f'(:n) \). Now, both are risk averse implying that

\[ v^{a}(\phi n) \neq v^{a}(0) \]  

(16)

ex-post, since, the marginal rate of substitution is a function of the labor supply when all are risk averse. For the proportion \( (1 - \phi) \) of individuals in unit \( j = a \) who do not work, there will be ex-post inefficiency, or

\[ E[\Delta'(0) + u'(c_{2a})z_{a}f'(\phi n)] \neq 0. \]  

(17)

For the proportion \( \phi \) of individuals in unit \( j = a \) who do work, there is ex-post efficiency, or

\[ E[\Delta'(\phi n) + u'(c_{2a})z_{a}f'(\phi n)] = 0. \]  

(18)

For all of \( j = b \), there is ex-post efficiency as in (5b). Thus, ex-ante efficiency is consistent with ex-post inefficiency at least for some in the population of \( j = a \).
4.2. Private information in individual preferences with heterogeneous types: no unemployment insurance

In the cases of Section 4.1, we assumed that there are complete markets for unemployment insurance so that a lottery holder can receive full wage in case of unemployment. As mentioned, this presumes that markets provide full insurance at actuarially fair prices. In this section, we assume that there are no insurance mechanisms available for $j=a$, i.e. there are incomplete markets in unemployment insurance. The individuals of unit $j=a$ are faced with idiosyncratic risk and use the capital market to insure against it.

We apply the same revelation mechanism except that the lottery contract specifies that the individual who does not work ex-post will not receive a payment; i.e. the terms of the contract are

- with probability $(1 - \phi) : n_a = 0$;
- with probability $\phi : n_a = n > 0$;
- $\phi$ gives no unemployment insurance to the holder.

In this case, ex-ante efficiency holds exactly as before, i.e. Pareto efficiency ex-ante (in expectations) holds, but ex-post allocations are inefficient and different from the cases in Section 4.1 above.

Once the lottery is revealed, the fraction $(1 - \phi)$ of individuals in unit $j=a$ who do not work will not be able to consume the same amount as the other fraction $\phi$ ex-post, since, with incomplete markets, they receive nothing in terms of wages in the second period, and have no labor income to trade. Therefore, the fraction $(1 - \phi)$ consumes out of the first period endowment and the fraction $\phi$ consumes more ex-post, or

$$c_{2a, \phi} = \frac{c_{2a, 1 - \phi}}{\Phi} > c_{2a, 1 - \phi}, \text{ full unemployment insurance}$$

Thus, in the case of incomplete markets for unemployment insurance there is ex-ante efficiency as before but there is ex-post inefficiency for some of $j=a$. The nature of the inefficiency includes the one discussed in expression (17) relating to the marginal disutility of labor, and in addition, it includes inefficiency in the marginal utility of consumption, since we obtain ex-post $u''(c_{2a, \phi}) \neq u''(c_{2a, 1 - \phi})$, i.e. consumption for all individuals in unit $j=a$ is not going to be identical ex-post. This implies that individuals use the capital market in different degrees.

We proceed to examine the implications of the different regimes of unemployment insurance and their effects on allocations, valuation and welfare.

5. Allocations, valuation and welfare with alternative regimes

We consider the effects of the alternative private information and unemployment insurance schemes on the valuation of consumption, work, and on the risk-free asset in the economy. The risk-free valuation of $1$ of saving in the first period is determined by expressions (5a) and (13a), where

$$R = 1/E[\beta (c_{2j})/u'(c_{1j})]$$

(19a)

gives the risk-free gross return on saving, denoted $R$, and saving is defined as

$$s_j = y - c_{1j} \text{ each } j.$$  

(19b)

The valuation of one unit of work in the first period is determined, from (5a,b), by

$$E[(\beta (c_{2j})/u'(c_{1j})) z_j f'(n_j)] = E[-\beta v'(n_j)/u'(c_{1j})] = p_{lj} \text{ all } j$$

(19c)

where the right-hand side denotes the price of one unit of labor next period, $p_{lj}$, valued in terms good in the first period. Eqs. (19a,b,c) are the main relationships that represent the asset valuation across units. For each case examined in Sections 3–4 regarding the nature of private information and market completeness of unemployment insurance, the set of efficiency relationships can be evaluated and comparisons across the different regimes governing private information and market completeness can be drawn.
Table 1

Efficient allocation and valuation with intertemporal trade under full information

<table>
<thead>
<tr>
<th></th>
<th>(\sigma_a=2.000)</th>
<th>(\gamma_b=0.750)</th>
<th>(\sigma_a=1.250)</th>
<th>(\gamma_b=0.750)</th>
<th>(\sigma_a=2.000)</th>
<th>(\gamma_b=0.250)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{ta})</td>
<td>1.081</td>
<td>1.456</td>
<td>1.249</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_{tb})</td>
<td>0.919</td>
<td>0.544</td>
<td>0.750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_{2a})</td>
<td>1.279</td>
<td>1.977</td>
<td>1.478</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_{2b})</td>
<td>1.087</td>
<td>0.739</td>
<td>0.888</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n_a)</td>
<td>0.084</td>
<td>0.034</td>
<td>0.041</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n_b)</td>
<td>0.561</td>
<td>0.902</td>
<td>0.671</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s_a)</td>
<td>-0.081</td>
<td>-0.456</td>
<td>-0.249</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s_b)</td>
<td>0.081</td>
<td>0.456</td>
<td>0.249</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\gamma_a)</td>
<td>0.574</td>
<td>0.335</td>
<td>0.372</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\gamma_b)</td>
<td>1.792</td>
<td>2.381</td>
<td>1.994</td>
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</tr>
<tr>
<td>(\epsilon_a)</td>
<td>2.713</td>
<td>8.848</td>
<td>5.621</td>
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<td></td>
</tr>
<tr>
<td>(\epsilon_b)</td>
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<td>0.348</td>
<td>0.217</td>
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<tr>
<td>(E[pl]_j)</td>
<td>2.038</td>
<td>2.413</td>
<td>2.468</td>
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<tr>
<td>(pl_a)</td>
<td>2.776</td>
<td>3.799</td>
<td>3.708</td>
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<tr>
<td>(pl_b)</td>
<td>1.300</td>
<td>1.027</td>
<td>1.211</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R=1/E[mrj])</td>
<td>1.473</td>
<td>1.543</td>
<td>1.473</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_a)</td>
<td>1.473</td>
<td>1.543</td>
<td>1.473</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_b)</td>
<td>1.473</td>
<td>1.543</td>
<td>1.473</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E[mrj])</td>
<td>0.679</td>
<td>0.648</td>
<td>0.679</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(mr_a)</td>
<td>0.679</td>
<td>0.648</td>
<td>0.679</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(mr_b)</td>
<td>0.679</td>
<td>0.648</td>
<td>0.679</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(\Omega)</td>
<td>-2.162</td>
<td>-1.095</td>
<td>-2.548</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Omega_a)</td>
<td>-1.868</td>
<td>-0.509</td>
<td>-1.540</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Omega_b)</td>
<td>-2.456</td>
<td>-1.680</td>
<td>-3.556</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\{\alpha = 0.6, \beta = 0.95, \delta = 2.5, \gamma_a = 0.75, \gamma_b = 0.25, \sigma = 2.000, \gamma_b = 0.250\}.

We pursue a quantitative approach in drawing the comparisons.\(^8\) Allocations and valuations are computed with \(J=2\), \(\alpha_j=\pi_j=1/2\) and \(z_j=2.534\) for \(j=a,b\) as in Table 1. At the benchmark set of Table 1, we use the plausible parameter choices seen in Section 3, that is

\{\alpha = 0.6, \beta = 0.95, \delta = 2.5, \gamma_b = \{0.75, 0.25\}, \sigma = \{2, 1.25\}, \gamma_1 = 1, \gamma = 6\}.

As noted above, Table 1 presents results for the basic set of parameter values. The first column is the basic economy in Section 3 where there is heterogeneity in risk aversion to labor supply by individuals \(j=b\), \(\gamma_b=0.75\), but full information. Individuals \(j=a\), are risk neutral in labor supply so work less and consume more in the first period thus borrowing more against future income. Individuals \(j=b\) are risk averse in labor supply so work more and consume less in the first period thus lending to individuals \(j=a\). Individuals \(j=a\) have a much higher elasticity of intratemporal substitution. The capital movement across units is small, of the order of one seventh of individual \(j=a\) output and more than one twentieth of individual \(j=b\) output. The market price of labor, according to the right-hand side of (19c), is also higher for \(j=a\). The marginal rates of substitution are identical by the capital market participation in (19a). The welfare of \(j=a\) is higher due to the higher consumption profile and lower work hours. Given the technology (2), we can infer from Table 1 that the return to fixed capital is higher for \(j=b\) since she works more and has a smaller residual while the reverse is observed for \(j=a\). Since \(j=a\) works less, her wage rate is much higher relative to \(j=b\).

The second and third columns provide sensitivity analysis. The second column is the case where the parameter governing intertemporal substitution, (1/) \(\sigma\), decreases from 2 to 1.25, indicating an increase in the elasticity of intertemporal substitution. The third column provides sensitivity to a change in the parameter governing the risk aversion in labor supply of individual \(j=b\), \(\gamma_b\) decreases to 0.25 indicating lower risk aversion for individual \(j=b\). The result relative to the first column is that the elasticity of intratemporal substitution of both individuals will increase. Due to the

\(^8\) Analytical comparisons are infeasible in this framework; see e.g. Leung (1995) for a case without private information where analytical results can be drawn.
nonlinearities in the model, the smaller the discrepancy in preferences towards labor supply when \( j=a \) is risk neutral, the larger is the extent of the uneven distribution of consumption and work across individuals.

In general, the sensitivity analysis in the second and third columns indicates that higher elasticity of intratemporal substitution for the risk neutral type \( j=a \) increases her consumption profile. For the risk averse individual \( j=b \) it decreases her consumption profile, making her work more. At the same time, it increases the volume of saving. Because \( j=b \) works slightly more, its own return to fixed capital increases, and the wage rate and price of labor decrease. In the case of a decrease in \( \sigma \) or higher elasticity of intertemporal substitution (second column), there is more capacity market activity (volume of savings increases) and the marginal rate of substitution decreases indicating an increase in the risk-free rate of return. For \( \gamma_b=0.25 \) (third column) the marginal rate of substitution is not significantly changed. Welfare improves substantially for all in the second column with higher elasticity of intertemporal substitution and more demand for consumption smoothing. However, welfare deteriorates for \( j=b \) in the case of her own lower risk aversion (third column) while improving slightly for \( j=a \).

The private information economy examined in Section 4.1 is the one where there is heterogeneity in risk aversion to labor supply by individuals \( j=b, \gamma_b=0.75 \), risk neutrality for individuals \( j=a, \gamma_a=0 \), and there is full unemployment insurance. The lottery contract makes individuals reveal truthfully and the allocations ex-ante and ex-post are identical to the economy with full information in Table 1 because full unemployment insurance allows all to smooth consumption identically regardless of working or not working in the second period. In the first column, the effective labor supply of the fraction \( \phi \) of individuals \( j=a \) who work is \( \phi_{n}=0.084 \), identical to the labor supply in the full information economy and the lottery is \( \phi=0.140 \). Similarly, in the second column the lottery is \( \phi=0.057 \), and in the third column it is \( \phi=0.068 \). The only difference is in the welfare of the fraction of individuals \( j=a \) who do not work, because ex-post they derive no disutility of labor and consume the same amount, thus welfare is higher for that share, \((1-\phi)\) of the population \( j=a \) who do not work.

The more important results are observed in Table 2, the ex-post allocation of the economy of Section 4.2, where the market regime is of no unemployment insurance. The ex-ante allocation would be identical to Table 1, Section 4.1, however once the lottery is revealed, the table shows an ex-post allocation radically different due to several inefficiencies. In the first column, we note the fraction \( \phi \) of \( j=a \) who work: They consume more in both periods than the fraction \( (1-\phi) \) that does not work. However, all individuals \( j=a \) become lenders in this allocation. The fraction \( (1-\phi) \) cannot consume out of second period output so it must use the capital market to save out of the first period endowment. The proportion \( \phi \) that works consumes more than the proportion \((1-\phi)\), but saves in the first period as well. As a result, individual \( j=b \) is better off since it works less and consumes more through borrowing in the first period. In this case, the trade in lotteries, together with open intertemporal capital markets, completely eliminate adverse selection. Relative to the full unemployment insurance regime (Table 1, first column), the volume of saving increases substantially for the fraction \((1-\phi)\) of type \( j=a \), to about one third of the output produced. In this allocation, the fraction \( \phi \) that actually works is much higher relative to the full unemployment insurance case. Hence, the average price of labor is much lower relative to the full insurance case, even though it is higher for type \( j=b \) because she works less.

The marginal rates of substitution are not identical across agents anymore because there are different degrees of capital market participation, i.e. the marginal rate of substitution of \( j=a \) is lower than \( j=b \), i.e. lack of full “risk sharing” due to private information. However, the key result is that, relative to the full unemployment insurance case, the marginal rate of substitution is lower across all units implying that the risk-free rate is higher in the economy without unemployment insurance, both for each group and on average across groups. The welfare of \( j=b \) is higher due to the higher consumption profile and lower work hours.

The second and third columns of Table 2 present sensitivity regarding \( \sigma \) and \( \gamma_b \) as in Table 1. The general effect is as before. We note an increase in the elasticity of intratemporal substitution of all types. Since now the types \( j=b \) are borrowers, the second column shows that their own consumption increases. The volume of capital market activity in the saving row also increases and the marginal rates of substitution decrease further showing a further rise in the risk-free rate of return.

The welfare results in the second column reveal that, in economies with high elasticity of intertemporal substitution (high demand for consumption smoothing), aggregate welfare is slightly larger than in economies without unemployment insurance. In particular, all \( j=a \) are worse off, but all \( j=b \) are better off bringing the average across groups slightly up. All other cases are consistent with the previous results of Diamond and Mirrlees (1978) who showed cases where it may be desirable to prevent private saving in favor of social insurance mechanisms. However, we show that that private saving is welfare improving relative to full unemployment insurance, at least in the case of high elasticity of intertemporal substitution (high demand for consumption smoothing). The third column, where risk aversion of \( j=b \) decreases, shows a
slight increase in the consumption pattern of $j=b$ and lower labor supply relative to the first column. In all cases adverse selection is fully mitigated.

The resulting comparison between an economy with full unemployment insurance, and no unemployment insurance (Tables 1 and 2), yields very sharp results regarding capital markets usage and returns. In economies with full unemployment insurance, capital markets work less towards mitigating and/or eliminating adverse selection due to private information in preferences regarding labor supply. This is because there is less demand for use of capital markets given that a lottery ticket holder knows that she will be receiving full salary whether working or not. In those economies the risk-free rate of return is lower. However, in economies with less than full unemployment insurance, or no unemployment insurance as in the example above, capital markets work much more towards mitigating and/or eliminating adverse selection due to private information in preferences regarding labor supply. In this case, there is much more demand for use of capital markets given that a lottery ticket holder knows that she will not be receiving full salary when the outcome is negative. Consequently, in those economies the risk-free rate of return is higher relative to the full unemployment insurance economy. This is a sharp hypothesis that can be, and should be tested with real world data. Countries with more generous unemployment insurance, for example Spain, should have lower use of capital markets and low risk-free returns whereas countries with less generous unemployment insurance, for example the U.S., should have more use of capital markets and a higher risk-free return.

6. Decentralization and competitive equilibrium

The allocations presented above, with a particular set of weights, can be decentralized in alternative ways. Given endowments, weights can be computed so that the optimal quantities and prices satisfy budget constraints, e.g. Backus
Another way would be to set up spot markets for goods and allow trade in assets and claims to the endowment by all in the economy, e.g. Lucas (1982). A competitive equilibrium for the economy studied in problem (4) would be given by prices and quantities that (i) maximize consumer’s utility subject to the consumer intertemporal budget constraint; (ii) maximize firms profit’s subject to technological constraints; and (iii) markets clear every period. Thus, for consumers, we have

$$\max_{\{c_1, c_2, n\}} E[\Omega(c_1, c_2, n)]$$

subject to

$$p_1c_1 + p_2c_2 - p_1y - wn \leq 0$$

(20a)

where \((p_1, p_2)\) are prices of the consumption good in periods 1 and 2 respectively and \(w\) is the wage in period 2. Optimal demands for goods, labor supply and the Lagrange multiplier associated with the budget constraint in (20a) will be functions of prices and the endowment as \(\{c_1, c_2, n, \gamma (p_1, p_2, w; y, \ldots)\}\). For firms, we have

$$\max_{\{n\}} E[p_2zf(n) - wn]$$

(21)

with optimal labor demand given by \(\{n(p_2, w; z, \ldots)\}\). Market clearing in every period requires: \(c_1 = y\) and \(c_2 = zf(n)\), so that a general equilibrium is characterized. Given the linear social welfare function in (4), the Lagrange multipliers of the planner’s problem, the functions \(\{q_1, q_2(\pi_j, \omega_j; y, \ldots)\}\), can be mapped into the multiplier of the decentralized equilibrium according to

$$\frac{\partial z}{\partial \omega_j} q_1(\pi_j, \omega_j; y, \ldots) = p_1\gamma(p_1, p_2, w; y, \ldots)$$

(22a)

$$\frac{\partial z}{\partial \omega_j} q_2(\pi_j, \omega_j; y, \ldots) = p_2\gamma(p_1, p_2, w; y, \ldots)$$

(22b)

Hence, the mapping between the efficient allocations and the equilibrium is established by Eqs. (22a,b). In the allocations studied, we fixed the weights \(\pi_j, \omega_j\) and the endowment \(y\), but those determine a set of (relative) market prices \((p_1, p_2, w)\) which can eventually support an equilibrium of the model, under the imposed distribution of the weights. Alternatively, given an equilibrium set of quantities and associated market (relative) prices \((p_1, p_2, w)\), and given the endowment \(y\), we can compute the weights \((\pi_j, \omega_j)\) that support an equilibrium of the model. The latter would involve the usual redistribution of endowments associated with the Second Fundamental Theorem. The key point of the analysis of this paper is that the restrictions in the parameter space that satisfy Proposition 1 are one example of an efficient allocation that can be plausibly an equilibrium of the model, under the given distribution of the weights. The alternative regimes of unemployment insurance do imply alternative sets of relative prices, and the ones studied above under a given distribution of the weights are also examples of allocations that can be plausibly an equilibrium of the model using mappings obtained similarly to (22a,b).9

7. Conclusions

We examined the effects of the presence of intertemporal capital markets in economies with private information regarding preferences towards labor supply. The introduction of another good at another date in an otherwise single period economy can have important effects on allocations and valuations. We show that a necessary and sufficient condition for an uneven distribution of consumption and work to occur under full information is that the type less risk averse towards labor supply must be a borrower in the first period. Then, we proceed to show that ex-post inefficiency can be consistent with ex-ante efficiency and analyze allocations under alternative regimes of unemployment

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9 Given the results in Rogerson (1988) and the additively separable utility assumed, e.g. Bianconi (2001), the ex-ante and ex-post efficient allocations can be mapped into equilibrium using expression (13d). Note also that by expressions (5a), (5b), (19a), (19c) and (22a), (22b), the return to saving and labor are mapped into the market prices so that an equilibrium with trade in assets and claims to the endowment can be replicated as well.
insurance. The main result is that the presence of capital markets can mitigate and/or reverse adverse selection based on heterogeneity in preferences towards labor supply, but this is particularly sensitive to the degree of unemployment insurance available for market participants. Our results suggest that in countries where unemployment insurance is generous capital markets have low usage and the risk-free rate of return is low. However, in countries where unemployment insurance is less generous, capital markets have more usage and the risk-free rate of return is higher. This is an important hypothesis that requires further empirical testing.

Earlier analytical work by Diamond and Mirrlees (1978) shows cases where it may be desirable to prevent private saving in favor of social insurance mechanisms. Our result is that generous unemployment insurance benefits could pose a barrier to private saving, thus affecting allocation and valuation. For example, we find that in the case of high intertemporal substitution (high consumption smoothing demand), aggregate welfare is larger in the economy without unemployment insurance than in the economy with unemployment insurance, thus indicating a potential benefit of private saving. More recently, Mulligan (2001) finds negligible quantitative aggregate implications of indivisibilities in labor supply. However, we show here that heterogeneity with private information together with alternative regimes for unemployment insurance can have rather important effects on allocations and valuation of assets across units, regions or countries.

Further research in the direction of introduction of real investment or the explicit consideration of moral hazard seems all worth pursuing.

References