Fiscal Policy and the Terms of Trade in an Analytical Two-Country Dynamic Model

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Abstract

This paper presents a two-country dynamic perfect foresight Ricardian model with wealth effects to study the relationship between government spending financed by alternative taxation, the terms of trade and welfare. An increase in domestic government spending financed by a distortionary capital income tax leads the real exchange rate initially to appreciate (a pure demand effect). But along the transitional path an intertemporal terms of trade effect (a supply side effect) operates and the real exchange rate depreciates to a steady state value ultimately higher relative to the initial equilibrium. The welfare of the domestic resident increases due to a reversed immiserizing growth effect.

Keywords: government spending, taxes, terms of trade

JEL Code: F4, H87, C6

1. Introduction

This paper presents an analytical two-country dynamic model with perfectly integrated capital markets to study movements in the terms of trade, capital accumulation, current account and welfare given an increase in government spending financed by alternative forms of taxation.

There has been intense empirical research trying to identify determinants of the terms of trade. Chinn and Johnston (1996) present a brief review of the literature and Chinn (1999, 2000) provide empirical evidence that government spending shocks lead to a deterioration of the terms of trade in the short to medium run, see also Froot and Rogoff (1991). This paper shows that the theoretical dynamic pattern of the terms of trade is consistent with these empirical results when an increase in government spending is financed by a distortionary capital income tax. However, we provide analytical support for the importance of the form of finance of government spending. In particular, when financed by capital income taxation, the pattern of the terms of trade exhibits a mixture of demand and supply effects. The current empirical evidence is not directed to identifying the effects of government spending under alternative forms of finance, but our analytical results suggest that some effort in this direction is warranted. Other authors such as Backus, Kehoe and Kydland (1994) find that government spending alone cannot reproduce the cross-correlation pattern of the terms of trade and the trade balance using a calibration approach. However, when coupled with technology shocks, theoretically comparable to distortionary capital income taxation,
their model produces a much better match consistent with the analytical results of this paper.

We find that an increase in domestic government spending financed by a distortionary capital income tax induces important intertemporal trade-offs. In a benchmark symmetric initial equilibrium, the real exchange rate initially appreciates (the pure demand effect), but along the transitional path the intertemporal terms of trade effect (the supply side effect) operates and the real exchange rate depreciates to a stationary value which is ultimately higher relative to the initial equilibrium (the real exchange rate depreciates across steady states). In this case, the trade balance deteriorates and the net asset position increases across steady states.

The government spending in our model does not provide direct utility for agents in the economy. However, we show that its ultimate welfare effect is positive when financed by capital income taxation. This is a surprising result, but the reason is simple. Under capital income tax finance, an increase in government spending induces a contraction of the domestic production possibility frontier, but the terms of trade effect allows the domestic resident to achieve a higher level of consumption in present discounted value terms, and welfare increases. Hence, we show that distortionary capital income taxation can have a positive welfare effect for reasons similar to a reverse immiserizing growth effect. In the case government spending is financed by a lump sum tax or domestic debt, by a tax on the consumption of the domestically produced good, or by a tax (tariff) on the consumption of the foreign good, the results above vanish.

The results obtained in this paper are in contrast to the one-good models of Turnovsky and Bianconi (1992), Bianconi (1995), and Bianconi and Turnovsky (1997). This is because, in this paper, there are initial wealth effects associated with real exchange rate responses to disturbances and capital does not relocate instantaneously. In the one-good case there are no wealth effects and capital reshuffles instantaneously.1

The paper is organized as follows. Section 2 presents the dynamic macroeconomic framework and Section 3 gives the general equilibrium, dynamics and stationary state. Section 4 presents the analytics of government spending financed by capital income taxation and its effect on the real exchange rate, capital accumulation, current account and welfare while Section 5 concludes. An appendix presents the dynamical systems.

2. Two-Country Macroeconomic Structure

The framework is founded on the optimization principles of a representative agent of the domestic and foreign country, a competitive framework for firms, and governments. Each country is assumed to specialize in the production of a distinct commodity. There are two goods: \( x \) is the good produced in the domestic (foreign) economy. A starred variable, \( x^* \) denotes the consumption of the domestic good by a foreign resident whereas an unstarrred variable denotes domestic consumption. In this context, \( \sigma \) denotes the relative price of the good produced in the foreign country, \( y \), in terms of the good produced in the domestic country, \( x \). Hence an increase (decrease) in \( \sigma \) denotes deterioration (improvement) of the domestic terms of trade or depreciation (appreciation) of the real exchange rate. \( \alpha \) denotes the marginal utility of wealth of the domestic resident in terms of the domestic good.
The representative household in the domestic country solves the intertemporal problem

$$\text{Max} \int_0^\infty U(x, y)(\exp - \beta t) \, dt \quad (1)$$

subject to the budget constraint expressed in terms of the domestic good as

$$x + \sigma y + \dot{k}^d + \sigma \dot{k}^{*d} = w + (1 - \tau) r k^d + (1 - \tau^*) r^* \sigma k^{*d} + T \quad (1a)$$

with $k_0^d > 0$ and $k_0^{*d} > 0$ given, where $\beta \in [0, \infty)$ is the domestic consumer subjective rate of time preference taken as given, $k^d(k^{*d})$ is the stock of domestic (foreign) capital held by the domestic resident, $w$ is the real wage income received by the domestic resident on the inelastically supplied unity labor, $r(r^*)$ is the real rate of return on domestic (foreign) capital. The tax structure is linear with $\tau \in [0, 1]$ denoting the tax rate on the return of domestic capital held by the domestic resident charged by the domestic government, $\tau^* \in [0, 1]$ denoting the tax rate on the return of foreign capital held by the domestic resident charged by the foreign government, and $T$ is a lump sum rebate/tax charged by the domestic government. The function $U$ is assumed to present the usual concavity properties in its two arguments. The household budget constraint consists of earnings on labor plus the after-tax returns on holdings of domestic and foreign capital plus any residual lump sum rebate (right-hand-side) to be spent on consumption of the domestic or foreign good or accumulation of domestic and foreign capital (left-hand-side).

Without loss of generality, I am going to consider the optimizing problem of the foreign household in terms of units of the domestic good. Hence, $\alpha^*$ be the marginal utility of wealth of the foreign resident in terms of the domestic good, with the ratio $\alpha^*/\sigma$ then yielding the marginal utility in terms of the foreign good. An interior solution for problem (1) and for the foreign consumer respectively is obtained by considering the set of first order conditions with respect to $x(x^*), y(y^*), k^d(k^{*d})$ and $k^{*d}(k^{*f})$ respectively given by

$$U_1(x, y) = \alpha \quad U_1^*(x^*, y^*) = \alpha^* \quad (2a)$$
$$U_2(x, y) = \sigma \alpha \quad U_2^*(x^*, y^*) = \sigma \alpha^* \quad (2b)$$

$$(1 - \tau) r = \beta - (\dot{\alpha}/\alpha) \quad (1 - \tau^*) r^* = \beta^* - (\dot{\alpha^*}/\alpha^*) \quad (2c)$$

$$(1 - \tau^*) r^* = (1 - \tau) r - (\dot{\sigma}/\sigma) \quad (2d)$$

Together with the transversality conditions

$$\lim_{t \to \infty} \alpha k^d(\exp - \beta t) = \lim_{t \to \infty} \alpha \sigma k^{*d}(\exp - \beta t) = \lim_{t \to \infty} \alpha^* k^f(\exp - \beta^* t) \quad (2e)$$
$$= \lim_{t \to \infty} \alpha^* \sigma k^{*f}(\exp - \beta^* t) = 0.$$  

Equations (2a,b) are marginal conditions for consumption of the domestic and foreign good, and (2c,d) are the usual arbitrage conditions equating the rates of returns of domestic and foreign capital to the return on domestic consumption including relative price changes.

The representative firm is assumed to operate with the usual constant returns to scale technology implying diminishing marginal products over the two inputs, capital and labor. Labor is assumed to be supplied inelastically and normalized to unity. The argmax of the
domestic and foreign profit functions $f(k) - rk - wI$ and $f(k^*) - r^*k^* - w^*l^*$ respectively imply

$$
\begin{align*}
ll f_k(k) &= r \\
f_k(k^*) &= r^* \\
w &= f(k) - f_k(k)k \\
w^* &= f(k^*) - f_k(k^*)k^*
\end{align*}
$$

(4a) (4b)

where the functional forms of the production functions are assumed to be the same across countries and $k = k^d + k^f$; $k^* = k^d + k^f$.

Domestic and foreign governments are assumed to operate under a source-based taxation scheme, see e.g., Turnovsky and Bianconi (1992). A balanced budget policy in both countries implies

$$
g + T = \tau rk \quad g^* + T^* = \tau^* r^* k^*
$$

(5)

where $g$ and $g^*$ are non utility-enhancing government consumption expenditures of the domestic and foreign country respectively. I assume that the domestic government only consumes the domestic good and the foreign government only consumes the foreign good.

The two distinct goods give goods market equilibrium conditions in each country as

$$
f(k) = x + x^* + \dot{k} + g \\
f(k^*) = y + y^* + \dot{k}^* + g^*
$$

(6)

The wealth of the domestic and foreign residents, $W$ and $W^*$ denoted in units of the domestic good are given by

$$
W = k^d + \sigma k^d \\
W^* = \sigma k^d + k^f
$$

(7)

Hence aggregate world wealth in units of the domestic good is given by $W + \sigma W^* = k + \sigma k^*$. The net foreign asset position of the domestic economy in units of the domestic good, $N$, is defined as

$$
N = \sigma k^d - k^f
$$

(8)

with respective wealth expressed as $W = k + N$, $W^* = \sigma k^* - N$; where $N > 0$ indicates a net credit for the domestic economy and a net debt to the foreign economy and vice-versa. From expression (8), *ceteris paribus*, an increase (decrease) in $\sigma$, i.e., a deterioration (improvement) of the domestic terms of trade increases (decreases) the value of the foreign assets held by the domestic resident thereby increasing (decreasing) the net foreign asset position of the domestic country. A formula for the change in the net foreign asset position or the current account balance is obtained by considering the time derivative of (8) together with the domestic household budget constraint (1a) and the domestic government budget constraint (5) yielding

$$
\dot{N} = f(k) - x - \sigma y - \dot{k} - g + (1 - \tau) f_k(k)N.
$$

(9)

This describes the accumulation of foreign assets by the domestic economy as the difference between domestic production and total domestic consumption plus (minus) the earnings (payments) on the stock of net foreign assets.
3. General Equilibrium, Dynamics and Stationary State

We consider benchmark equilibrium evaluated at no distortionary taxes, where utility is log linear, additively separable in both goods, and identical across countries with equal weights on preferences for the domestic and foreign goods, see e.g., Stockman and Tesar (1992). Technology is Cobb-Douglas and also identical across countries. Initial government spending is positive and identical across countries, and the initial equilibrium is one where the domestic country is a net creditor, \( N > 0 \). In this benchmark case, the initial equilibrium is symmetric in terms of the relative prices, or \( \sigma = 1 \), and the only difference between the two countries is the consumption profiles of the goods. \(^2\) From (2a,b), the static solutions for the short run consumption demands are of the form

\[
x = x(\alpha) \quad x_a < 0 \\
x^* = x^*(\alpha^*) \quad x^*_a < 0
\]  

(10a)

\[
y = y(\alpha, \sigma) \\
\quad y_a < 0, \quad y_\sigma < 0 \\
y^* = y^*(\alpha^*, \sigma) \quad y^*_a < 0, \quad y^*_\sigma < 0
\]  

(10b)

These describe the usual negative relationships between consumption levels and their marginal utilities. The effect of an increase (deterioration) in the terms of trade, \( \sigma \), is to reduce the quantity of the foreign good consumed by the domestic resident as well as the quantity of the foreign good consumed by the foreign resident since this is denominated in units of the domestic good. The dynamic system in prices, \( \sigma, \alpha, \alpha^* \), and quantities \( k, k^* \) and \( N \) is described by the equations (2c,d), (6) and (9) respectively, given initial quantities, \( k_0 > 0 \) and \( k^*_0 > 0 \), and given an initial net asset position, \( N_0 \). The equilibrium is closed by considering the usual condition that prevents any of the two economies from running an infinite amount of liabilities, or

\[
\lim_{t \to \infty} N(t) \left\{ \exp - \int_0^t (1 - \tau) f_k[k(s)] ds \right\} = 0.
\]  

(11)

The fixed rates of time preference in both countries are equated at all points in time, \( \beta = \beta^* \forall t \), guaranteeing the existence of an interior stationary equilibrium. This implies that the marginal utilities of wealth across countries are related by a constant \( \mu > 0 \) at all points in time except on initial impact of disturbances to the stationary equilibrium, \(^3\)

\[
\alpha^* = \mu \alpha, \forall t^{+,-}.
\]  

(12)

A direct consequence of (12) and the assumption of additively separable preferences is that the dynamic adjustment becomes recursive. We obtain a homogeneous two-variable system in \( \alpha \) and \( k \), and using this result we analyze a non-homogeneous two-variable system in \( \sigma \) and \( k^* \). The saddlepoint defined by (A.1)–(A.2) in the appendix yields time paths for the stable solution for \( k \) and \( \alpha \), for every time \( t \), given by

\[
k(t) = k^5 + (k_0 - k^5)(\exp \lambda_1 t) \\
\alpha(t) = \alpha^5 + \phi(\lambda_1)(k_0 - k^5)(\exp \lambda_1 t)
\]  

(13a)

where \( \lambda_1 = \{\beta - [\beta^2 + 4\alpha^3 f_k(x_a + x^*_a \mu)]^{1/2}/2 < 0 \) is the stable root of the characteristic equation of (A.1) and \( \phi(\lambda_1) = -\alpha^3 f_k/k_1 = (x_a + x^*_a \mu)/(\beta - \lambda_1) < 0 \). In the incomplete markets model of Turnovsky (1997), physical capital cannot cross borders and the dynamical system is homogeneous and can be solved independently in the two countries. The solution
(13a,b) guarantees a stable dynamic adjustment to the stationary equilibrium with \( k_0 \) initially predetermined and \( \sigma(0) \) free to take initial jumps. The implied stable manifold between \( k \) and \( \sigma \) is negatively sloped as in any regular optimal growth model. Given (12)–(13b), the time path of the marginal utility of wealth of the foreign resident is given by

\[
\alpha^*(t) = \sigma^S + \mu \phi(\lambda_1)(k_0 - k^S)(\exp \lambda_1 t).
\]  
(13c)

We can now use (13a) and consider the dynamics of the relative price, \( \sigma \), and the foreign capital stock, \( k^* \), by linearizing (2d) and (6) in the neighborhood of stationary values denoted by \( \sigma^S \) and \( k^S \). From (A.3)–(A.5), the final solution for the time paths of \( \sigma \) and \( k^* \), for every time \( t \), consists of the sum of the homogeneous and the non-homogeneous parts, and is given by

\[
\sigma(t) = \sigma^S + \left(a^*_{12}/\lambda_2\right)[(k^*_0 - k^S) - (k_0 - k^S)](\exp \lambda_2 t)
- \left[a^*_{12}(b^*_{21} + \lambda_1 - \beta)/[\lambda_1(\lambda_1 - \beta) - a^*_{12}a^*_{21}]\right](k_0 - k^S)(\exp \lambda_1 t).
\]
(13d)

\[
k^*(t) = k^S + (k^*_0 - k^S)(\exp \lambda_2 t) + (k_0 - k^S)[(\exp \lambda_1 t) - (\exp \lambda_2 t)].
\]
(13e)

The two stable roots determine the speed of adjustment of the endogenous variables and, in the benchmark symmetric equilibrium, we obtain \( 0 > \lambda_1 > \lambda_2 \). The last dynamic equation describes the evolution of the net foreign asset position. Linearizing equation (9) in the neighborhood of the stationary equilibrium, \( N^S \), and then substituting for the equilibrium paths from equations (13) yields a single differential equation in \( N \) with solution that satisfies the solvency condition (11) given by

\[
N(t) = N^S - (k_0 - k^S)[\psi_1(\exp \lambda_1 t) + \psi_2(\exp \lambda_2 t) - (\exp \lambda_2 t)]
- \psi_2(k^*_0 - k^S)(\exp \lambda_2 t)
\]
(13f)

where

\[
\psi_1 \equiv \{-b^*_{21}y(1 + \epsilon)[\lambda_1(\lambda_1 - \beta) + \lambda_2(\lambda_2 - \beta)]/a^*_{21}(\lambda_1 - \beta)\}[\lambda_1(\lambda_1 - \beta) - a^*_{12}a^*_{21}]
- [(\lambda_1 + \sigma^S \gamma_0)/(\lambda_1 + \sigma^S \gamma_0 + \mu \phi(\lambda_1))] + [(\lambda_1 - \beta - f_{kk} N^S)/(\lambda_1 - \beta)];
\]

and

\[
\psi_2 \equiv y(1 + \epsilon)/a^*_{21} \text{ with } \epsilon \equiv \gamma_0 \sigma^S/\gamma < 0 \text{ being the elasticity of domestic demand for the foreign good with respect to changes in the real exchange rate. The elasticity parameter basically determines how movements in the real exchange rate affect the domestic demand for imports. In the benchmark equilibrium, the elasticity becomes minus one, or } \epsilon = -1 \text{ implying that the domestic demand for the foreign good is unit elastic, implying } \psi_2 = 0 \text{ and } \psi_1 > 0 \text{. In summary, the dynamic adjustment from an initial stationary equilibrium, } k_0 > 0, \text{ for } k_0 > 0 \text{ and } N_0, \text{ for } k, \alpha^*, \sigma, k^*, \text{ and } N \text{ is described by equations (13a–f) respectively where } k \text{ and } k^* \text{ are predetermined.}
\]

Given the initial conditions and the dynamic adjustment, the steady state is consistent with

\[
\dot{k} = \dot{\alpha} = \dot{\sigma} = \dot{k}^* = \dot{N} = 0
\]
(14)
and described by the set of equations

\[
(1 - \tau)f_x(k^S) = (1 - \tau^*)f_x(k^{*S}) = \beta
\]
\[
f(k^S) = x(\alpha^S) + x^*(\alpha^{*S}) + g; \quad f(k^{*S}) = y(\alpha^S, \sigma^S) + y^*(\alpha^{*S}, \sigma^{*S}) + g^*
\]
\[
\beta N^S = \sigma^S y(\alpha^S, \sigma^S) - x^*(\alpha^{*S}) \tag{15c}
\]
\[
N^S - N_0 = \psi_1(k_0 - k^S) + \psi_2(k^S - k^{*S}) \tag{15d}
\]
\[
\alpha^{*S} = \mu \alpha^S. \tag{15e}
\]

These solve for \(k^S, k^{*S}, \alpha^S, \alpha^{*S}, \sigma^S, N^S\), and \(\mu\), as an implicit function of the tax rates and government spending parameters, \(\tau, \tau^*, g, \) and \(g^*\) with the lump sum rebates adjusting to balance the budgets in both countries, and the given initial conditions, \(k_0 > 0, k^*_0 > 0,\) and \(N_0^*.\)

The trade balance of the domestic country is easily defined as domestic production minus absorption. Using (15c), the stationary trade balance is given by \(TB^S = -\beta N^S\) and using (6) its time path is

\[
TB(t) = -\beta N^S + [\beta - \lambda_1 + \phi(\lambda_1)](k_0 - k^S)(\exp \lambda_1 t) - y(1 + \epsilon)(\sigma(t) - \sigma^S). \tag{16}
\]

In the stationary state, the domestic trade balance is just the negative of the capitalized stock of net credit of the domestic economy. Over time, the domestic trade balance depends on the domestic capital accumulation and the real exchange rate dynamics as well. The effect of the domestic capital accumulation is weighted by the term \(\beta - \lambda_1 + \phi(\lambda_1) > 0\) which is unambiguously positive. Hence as the domestic capital stock decreases (increases) along the transitional path, the domestic economy accumulates a trade surplus (deficit). The real exchange rate dynamic path is weighted by the term \(-y(1 + \epsilon)\) that depends on the magnitude of the elasticity of domestic demand for the foreign good. If \(\epsilon < -1\) (elastic demand), then \(-y(1 + \epsilon) > 0\) and an increase or devaluation (decrease or appreciation) in the real exchange rate along the transitional path leads to a trade balance surplus (deficit). If \(\epsilon > -1\) (the demand is inelastic), then \(-y(1 + \epsilon) < 0\) and an increase (decrease) in the real exchange rate along the transitional path leads to a trade balance deficit (surplus). In the benchmark symmetric equilibrium, \(\epsilon = -1\) and the dynamics of the real exchange rate along the transitional path do not affect the trade balance. Finally, the discounted utility of each individual in equation (1) can be suitably used as a measure of welfare. This is obtained as a first order linear approximation of the functions \(U\) and \(U^*\) in the neighborhood of a stationary equilibrium, and then substituting the appropriate dynamic paths from (13) and integrating, see e.g., Brock and Turnovsky (1993). The resulting welfare denoted by \(Z\) and \(Z^*\) in units of the domestic good for the domestic and foreign residents respectively are given by

\[
Z = \{U[x(\alpha^S), y(\alpha^S, \sigma^S)]/\beta] + z_1(k_0 - k^S) + z_2(k^S - k^{*S}) \tag{17a}
\]
\[
Z^* = \{U^*[x^*(\alpha^{*S}), y^*(\alpha^{*S}, \sigma^{*S})]/\beta] + z_1^*(k_0 - k^S) + z_2^*(k^S - k^{*S}) \tag{17b}
\]
where

\[ z_1 = \{(U_1 x_a + U_2 y_a) \phi(\lambda_1) U_2 y_a a_{12}^* (b_{21}^* + \lambda_1 - \beta) / \left[ \lambda_1 (\lambda_1 - \beta) - a_{12}^* / a_{21}^* \right] (\beta - \lambda_1)\} - z_2; \]

\[ z_1^* = \{(U_1^* x_a^* + U_2^* y_a^*) \mu_0 \phi(\lambda_1^*) U_2^* y_a^* a_{12}^* (b_{21}^* + \lambda_1^* - \beta) / \left[ \lambda_1^* (\lambda_1^* - \beta) - a_{12}^* / a_{21}^* \right] (\beta - \lambda_1^*)\} - z_2^*; \]

\[ z_2 = -U_2 y_a a_{12}^*/\lambda_2 (\beta - \lambda_2) < 0; \quad z_2^* = -U_2^* y_a^* a_{12}^*/\lambda_2 (\beta - \lambda_2) < 0. \]

In the benchmark equilibrium, both \( z_1 < 0 \) and \( z_1^* < 0 \) hold. Hence, as domestic and/or foreign capital decrease (increase) along the transitional path, domestic and foreign welfare decrease (increase) since the decumulation of capital leads to lower long run consumption pattern.

4. Fiscal Policy, the Terms of Trade and Welfare

The initial impact of a disturbance to the stationary equilibrium can be obtained by first noting that \( k_0^* \), \( k_0^* \) are predetermined variables implying that \( d k_0 = d k_0^* = 0 \). There is a wealth effect on impact derived from the initial jump in the relative price. This initial wealth effect is a pure scale effect, hence aggregate world wealth responds instantaneously but, the share of each country in total world wealth is initially fixed. It follows that given a disturbance to the stationary equilibrium, the initial wealth effect is distributed across countries according to

\[ d W_0 = \delta \sigma^S d W_0^* \] (18a)

where \( \delta \equiv -(k_0^* / W_0^*) / (k_0 / W_0) < 0 \) is a constant, implying that the initial effects on the net asset position and the relative price are related by\(^5\)

\[ d N_0 = \left[ \delta (1 + \delta) \right] (N^S / \sigma^S) d \sigma (0). \] (18b)

Equation (18b) describes the relationship between the net asset position and the real exchange rate, when the economy is subject to a disturbance. This relationship depends on the initial conditions of the two economies through the parameter \( \delta < 0 \) and on the creditor/debtor status of the domestic/foreign country through \( N \). However, these are interrelated. In the presence of disturbances, \( d N_0 \) and \( d \sigma (0) \) are positively related for any \( d N_0 \neq d \sigma (0) \neq 0 \). For the proof, note that \( \delta \in (-\infty, 0) \).

(i) When \( \delta \in (-1, 0) \), then \( (k_0 / W_0) > (k_0^* / W_0^*) \) or the initial fraction of domestic capital in domestic wealth is greater than the initial fraction of foreign capital in foreign wealth, implying that \( [\delta (1 + \delta)] < 0 \); but by the definitions of \( W_0 \) and \( W_0^* \), \( (k_0 / W_0) > (k_0^* / W_0^*) \) implies that \( N_0 < 0 \), or the domestic country is initially a net debtor, hence \( d N_0 \) and \( d \sigma (0) \) are positively related.

(ii) When \( \delta \in (-\infty, -1) \), then the initial fraction of domestic capital in domestic wealth is less than the initial fraction of foreign capital in foreign wealth implying that \( [\delta (1 + \delta)] > 0 \); by the definitions of \( W_0 \) and \( W_0^* \), \( (k_0 / W_0) < (k_0^* / W_0^*) \) implies that \( N_0 > 0 \), or the domestic country is initially a net creditor, hence \( d N_0 \) and \( d \sigma (0) \) are positively related.

(iii) For \( \delta \to -1 \), it implies that the fractions are identical and \( [\delta (1 + \delta)] \to +\infty \) with \( d N_0 = d \sigma (0) = 0 \).
therefore, \( dN_0 \) and \( d\sigma(0) \) always move in the same direction, and since in the benchmark equilibrium, \( N^S > 0 \), we have that \( [\delta/(1 + \delta)] > 0.6 \).

**A. Increase in Domestic Government Spending Financed by Domestic Lump Sum Tax**

First, in a stationary equilibrium where there are no distortionary taxes, a marginal increase in \( g \) financed by \( T \) implies that \( dg = -dT \) from the government budget constraint (5). In this case, \( g \) has no steady state effects on the domestic and foreign capital stocks as may be seen from (15a), hence the capital stocks remain constant. When evaluated in the absence of distortionary taxes, this model satisfies the Ricardian property such that debt and lump sum tax finance are equivalent and there are no transitional dynamics in any of the endogenous variables for the government spending disturbance financed by lump sum tax. Consider (18b) to obtain

\[
dN_0/dg \bigg|_{dT} = \left[ \delta/(1 + \delta) \right] (N^S/\sigma^S) d\sigma(0)/dg \bigg|_{dT}, \tag{19a}
\]

by (13d) evaluated at \( t = 0 \) and (15d) I find that \( d\sigma(0)/dg = d\sigma^S/g \) and

\[
dN_0/dg \bigg|_{dT} = \left[ \delta/(1 + \delta) \right] (N^S/\sigma^S) d\sigma^S/g \bigg|_{dT}, \tag{19b}
\]

and there are no transitional dynamics.\(^7\)

Using equations (19) above and the stationary conditions (15b–d) I obtain solutions for the effects on \( \alpha^S, \alpha^{S*}, \sigma^S, \) and \( N^S \) given by

\[
d\alpha^S/dg \bigg|_{dT} = (x^*_a a^{*}_{21} - c_{11} y^*_a)/(c_{11} c_{22} - x^*_a a^{*}_{21} c_{12}) \tag{20a}
\]

\[
d\alpha^{S*}/dg \bigg|_{dT} = y_a (\sigma^S a^{*}_{21} + c_{11})/(c_{11} c_{22} - x^*_a a^{*}_{21} c_{12}) \tag{20b}
\]

\[
d\sigma^S/dg \bigg|_{dT} = y_a c^*_{12}/(c_{11} c_{22} - x^*_a a^{*}_{21} c_{12}) \tag{20c}
\]

\[
dN^S/dg \bigg|_{dT} = \left[ \delta/(1 + \delta) \right] (N^S/\sigma^S) y_a c^*_{12}/(c_{11} c_{22} - x^*_a a^{*}_{21} c_{12}) \tag{20d}
\]

where

\[
c_{11} = y(1 + \epsilon) - \beta[\delta/(1 + \delta)](N^S/\sigma^S) \quad c_{12} = x_a + \sigma^S y_a < 0
\]

\[
c^*_{12} = x^*_a + \sigma^S y^*_a < 0 \quad c_{22} = x_a y^*_a - x^*_a y_a.
\]

**B. Increase in Domestic Government Spending Financed by Domestic Capital Income Tax**

In the stationary equilibrium where there are no distortionary taxes, an increase in \( g \) financed by raising \( \tau \) implies that \( dg = -\beta k^S d\tau \) from the government budget constraint (5). From the stationary equilibrium conditions (15a) we note that \( dk^S/d\tau = \beta/f_k < 0 \) and \( dk^{S*}/d\tau = 0 \). Hence, the overall effect of \( g \) financed by \( \tau \) in the stationary domestic capital stock is given by \( dk^S/d\tau = \beta^S k^S/f_k < 0 \) and it follows that there are transitional dynamics in the endogenous variables for a permanent government spending shock financed by a capital
income tax. To show this note from (18b) that
\[\frac{dN_0}{d\gamma} \bigg|_{d\xi} = \left[\frac{\delta}{1+\delta}\right](N^S/\alpha^S) \frac{d\sigma(0)}{d\gamma} \bigg|_{d\xi}\] (21a)
and by (13d) evaluated at \( t = 0 \) and (15d) I find that
\[\frac{d\sigma(0)}{d\gamma} \bigg|_{d\xi} = \frac{d\sigma^S}{d\gamma} \bigg|_{d\xi} + \left(a_{12}\left[1 + \lambda_2(b_{21} + \lambda_1 - \beta)\right]\right)\]
\[\frac{\lambda_2[\lambda_1(1 - \beta) - a_{12}a_{21}^\alpha]}{\lambda_2[\lambda_1(1 - \beta) - a_{12}a_{21}^\alpha]} \frac{dT}{d\gamma}\] (21b)
\[\frac{dN_0}{d\gamma} \bigg|_{d\xi} = \frac{dN^S}{d\gamma} \bigg|_{d\xi} + \psi_1 dk^S/d\gamma,\] (21c)
and the transitional dynamics in the other variables follow directly. Upon direct substitution
of (21b,c) into (21a), I obtain
\[\frac{d\sigma^S}{d\gamma} \bigg|_{d\xi}\left[\frac{\delta}{1+\delta}\right](N^S/\alpha^S) - \frac{dN^S}{d\gamma} \bigg|_{d\xi} = (dk^S/d\gamma)c_{21}\] (21d)
where
\[c_{21} = (\psi_1 + [\delta/(1+\delta)](N^S/\alpha^S)[a_{12}^\alpha\left[1 - \lambda_2(b_{21} + \lambda_1 - \beta)\right]/[\lambda_2[\lambda_1(1 - \beta) - a_{12}a_{21}^\alpha]])].\]
Evaluated at the benchmark equilibrium, I can show that
\[a_{12}^\alpha\left[1 + \lambda_2(b_{21} + \lambda_1 - \beta)\right]/[\lambda_2[\lambda_1(1 - \beta) - a_{12}a_{21}^\alpha]] > 0\]
in expression (21b) and thus \(c_{21} > 0\). Hence, combining the expressions above with the stationary conditions (15b–d) yield solutions for the steady state effects on \(\alpha^S, \alpha^S, \sigma^S, \) and \(N^S\) given by
\[\frac{d\alpha^S}{d\gamma} \bigg|_{d\xi} = \frac{\beta(c_{11}c_{21}^2 - x_a^\alpha a_{21}^\alpha c_{11})}{d\gamma}\] (22a)
\[\frac{d\alpha^S}{d\gamma} \bigg|_{d\xi} = \frac{\beta(c_{11}c_{22} - x_a^\alpha a_{22}^\alpha c_{11})}{d\gamma}\] (22b)
\[\frac{d\alpha^S}{d\gamma} \bigg|_{d\xi} = \frac{\beta(c_{12} + c_{21}c_{22} - x_a^\alpha a_{21}^\alpha c_{12})}{d\gamma}\] (22c)
\[\frac{dN^S}{d\gamma} \bigg|_{d\xi} = \frac{\beta(c_{12} + c_{21}c_{22} - x_a^\alpha a_{21}^\alpha c_{12})}{d\gamma}\] (22d)
The terms in keys multiplying the induced effect in the stationary domestic capital stock,
\(dk^S/d\gamma\), in the expressions above represent the distortions introduced due to the financing
by the distortionary capital income tax relative to the non distortionary lump-sum tax case.

**Proposition 1** In the symmetric benchmark equilibrium:
\[\text{sgn}(d\alpha^S/d\gamma) = \text{sgn}[-(dk^S/d\gamma) - y_a] > 0;\]
\[\text{sgn}[(d\sigma(0) - d\alpha^S)/d\gamma] = \text{sgn}[-(\alpha^S(x_a + x_a^\mu) - \sigma^S(y_a + y_a^\mu))] < 0;\]
\[\text{sgn}(d\sigma(0)/d\gamma) < 0.\]

The proof follows algebraically from (21b) and (22c) using \(x_a^\gamma y_a = x_a^\gamma y_a\) and the character-
istic equation of (A.1). In particular, the first expression is due to \((dk^S/d\gamma) > y_a\) and the
second expression is because of the ordering of the two stable eigenvalues \(0 > \lambda_1 > \lambda_2.\)
In the symmetric benchmark case, all the distortionary effects in (22a–d) outweigh the non-distortionary effects. Hence, $\frac{d\alpha^s}{dg}|_{dt} < 0$, implying that $\frac{d\mu}{dg}|_{dt} > 0$, and $\frac{dN^S}{dg}|_{dt} > 0$. Also, from (21a), $\frac{dN_0}{dg}|_{dt} < 0$, and from (21c), $(\frac{dN_0 - dN^S}{dg}|_{dt} < 0$. Figure 1(a,b) illustrates the dynamic adjustment for the symmetric benchmark case in the $[\alpha, k, \sigma, N]$ space. The initial equilibrium is at point $A$. At the time of the disturbance, the domestic marginal utility initially decreases (the foreign marginal utility decreases as well), together with an initial fall (appreciation) of the real exchange rate and the consequent fall in the net asset position, all to point $A'$. Along the transition, since the
capital income tax has been implemented, domestic investment declines creating an excess supply of domestic output channeled through the acquisition of foreign assets, as domestic consumption declines as well. Thus, the real exchange rate depreciates (or the terms of trade deteriorate) in the transition and the domestic economy runs a current account surplus mirrored by the associated trade balance deficit, converging to the new stationary equilibrium at point $B$. Intuitively, the increase in government spending has an instantaneous positive domestic demand effect since the domestic capital stock is initially fixed, and the terms of trade instantaneously improve. However, along the transition the supply side effect induced by the increase in the capital income tax takes charge leading to the deterioration of the terms of trade along the transitional path, and the eventual long run devaluation of the real exchange rate. The time path of the real exchange rate resembles an S-shaped pattern. The instantaneous temporal terms of trade effect, or the pure demand effect, trade-off with the intertemporal terms of trade effect based on the transitory movements of the real interest rate, or the supply side effect. Initially, the increase in the domestic capital income tax assigns a negative gap between the after-tax domestic real rate of return and the world discount rate, which remains equated to the foreign real rate of return. Along the transitional path, this real interest rate differential induces an outflow of domestic capital into foreign assets and the domestic country runs a current account surplus, the domestic capital stock declines and the after-tax real rate of return increases to close the gap with the world discount rate. Hence, while there is a permanent depreciation of the real exchange rate, there is only a transitory change in the after-tax domestic real interest rate. The stationary effect on the trade balance is given by

$$dTB^S/dg|_{dt} = -\beta dN^S/dg|_{dt}$$

(22e)

and the initial effect is obtained from (16) as

$$dTB(0)/dg|_{dt} = -\beta dN^S/dg|_{dt} - [\beta - \lambda_1 + x_0\phi(\lambda_1)]dk^S/dg - y(1 + \epsilon)[d\sigma(0)/dg|_{dt} - d\sigma^S/dg|_{dt}].$$

(22f)

In the benchmark scenario, the trade balance has the reverse dynamic path relative to the net asset position, or from (22e,f) I obtain

$$[dTB(0) - dTB^S]/dg|_{dt} = -[\beta - \lambda_1 + x_0\phi(\lambda_1)]dk^S/dg > 0.$$

While the trade balance initially improves, $dTB(0)/dg|_{dt} > 0$, the net asset position initially deteriorates. Along the transitional path, the trade balance deteriorates with the steady state effect being negative, $dTB^S/dg|_{dt} < 0$, while the reverse occurs with the net asset position. The welfare effects of the change in government spending are given by

$$dZ/dg|_{dt} = [(U_1x_0 + U_2y_0)/\beta]d\alpha^S/dg|_{dt} + (U_2y_0/\beta) d\sigma^S/dg|_{dt} - z_1 dk^S/dg]$$

(22g)

and

$$dZ^*/dg|_{dt} = [(U_1^*x_0^* + U_2^*y_0^*)/\beta]d\alpha^{S*}/dg|_{dt} + (U_2^*y_0^*/\beta) d\sigma^{S*}/dg|_{dt} - z_1^* dk^{S*}/dg].$$

(22h)

**Proposition 2** In the benchmark symmetric equilibrium with no initial distortions, an increase in domestic government spending financed by a capital income tax is welfare improving for the domestic country.
The proof is simple. The domestic welfare effect in (22g) has two components of opposite sign. The pure consumption effect is given by the term \(\left(\frac{U_1 x_\alpha + U_2 y_\alpha}{\beta}\right) \frac{d\sigma^\delta}{d\tau} > 0;\) which has a positive impact on domestic welfare because the distortion introduced by \(\tau\) leads to an increase in domestic consumption of the domestic and foreign good. There is the sum of the real exchange rate and capital stock effects, \(\left(\frac{U_2 y_\sigma}{\beta}\right) \frac{d\sigma}{d\tau} > z_1 dk^\delta > 0;\) which are both negative because the long run depreciation of the real exchange rate decreases the domestic demand for the foreign produced good, and the lower long run capital stock is consistent with lower long run aggregate consumption. In the symmetric benchmark equilibrium, the pure consumption effect outweighs the sum of the real exchange rate and capital stock effects, and the welfare of the domestic resident increases while the domestic trade balance deteriorates. Since in the benchmark case the domestic economy is a net creditor, \(N > 0, \frac{dZ}{d\tau} > \frac{dZ^*}{d\tau} > 0,\) or welfare improves in both countries but more in the domestic country relative to the foreign country. However, if the domestic country is a net debtor, \(N < 0,\) then \(\frac{dZ}{d\tau} > \frac{dZ^*}{d\tau} > 0,\) or welfare improves more in the foreign country relative to the domestic country. Note that in both cases, the welfare of the foreign resident is denominated in terms of the domestic good. If welfare of the foreign resident is denominated in terms of the foreign good, it falls in both cases but less when the foreign country is a net creditor. The result in Proposition 2 is surprising given that we start from a position of zero distortions, i.e., it is a first order effect. If the result had been obtained from an initial position of positive distortion, an increase in welfare would simply imply a standard second-best paradox: given the initial distortion, an additional one may undo some of the existing inefficiency. However, we start from zero distortions. How can we obtain this result? The answer is closely related to the well-known mechanism of immiserizing growth in real trade models with large countries under free trade, see e.g., Bhagwati (1968). In that case, an outward shift in the production possibility frontier of a nation may induce a change in the terms of trade that will give a lower consumption bundle and thus lower welfare for the resident of that nation. In the case of Proposition 2, an increase in government spending financed by a capital income tax induces an inward shift in the production possibility frontier of the domestic country. This is coupled with a change in the terms of trade that yields a higher consumption bundle, in present discounted value, for both nations in terms of the domestic good. Hence, the discounted utility increases for the domestic country, i.e., a reversed immiserizing growth effect. Thus, variable terms of trade are crucial for welfare analysis in large open economies.

5. Conclusions

This paper has presented a two-country dynamic perfect foresight Ricardian model with inelastic labor supply and wealth effects to study the relationship between government spending financed by capital income taxation, the terms of trade and welfare. A marginal increase in domestic government spending financed by a distortionary capital income tax under an initial symmetric benchmark equilibrium leads to: (i) the real exchange rate initially appreciates (the pure demand effect), but along the transitional path the intertemporal terms of trade effect (a supply side effect) operates and the real exchange rate depreciates to a steady state value which is ultimately higher relative to the initial equilibrium, the real
exchange rate depreciates across steady states; (ii) welfare of the domestic resident increases due to a reversed immiserizing growth effect.

In this paper, technical restrictions were imposed for the sake of analytical tractability. In particular, it is worth further pursuing the case where domestic government spending includes foreign produced goods thus making the role of the elasticity of the domestic demand for the foreign good a more critical parameter, and including variable labor is also a fruitful avenue.

Appendix

The homogenous two-variable system of differential equations in the variables $\alpha$ and $k$, is described by equations (2c) and (6) together with the equilibrium conditions for firms (4), the static relationships (10) and the ratio of the marginal utilities (12). Linearizing this system in the neighborhood of a stationary equilibrium, whose existence is guaranteed by (12), and denoting stationary values by $\alpha^S$ and $k^S$, gives

$$\dot{D} = AD$$  \hspace{1cm} (A.1)

where $D_0 = \{\alpha - \alpha^S, (k - k^S)\}$ is a row vector and $A$ is a $2 \times 2$ matrix of coefficients which upon evaluation at an initial stationary equilibrium of zero distortionary taxes are given by

$$a_{11} = 0 \hspace{1cm} a_{12} = -\alpha^S f_{kk} > 0$$

$$a_{21} = -\kappa_\gamma - \kappa^*_\alpha \mu > 0 \hspace{1cm} a_{22} = \beta > 0.$$  \hspace{1cm} (A.2)

The characteristic equation of (A.1) has a negative determinant implying that the roots are real and have opposite sign. I can now use (13a) and consider the dynamics of the relative price, $\sigma$, and the foreign capital stock, $k^*$, by linearizing (2d) and (6) in the neighborhood of stationary values denoted by $\sigma^S$ and $k^S$. This gives the non-homogeneous two-variable system of differential equations denoted by

$$\dot{D}^* = A^*D^* + B^*E^*$$  \hspace{1cm} (A.3)

where $D^0 = \{(\sigma - \sigma^S), (k^* - k^S)\}$ is a row vector, $A^*$ is a $2 \times 2$ matrix of coefficients which upon evaluation at an initial stationary equilibrium of zero distortionary taxes are given by

$$a_{11}^* = 0 \hspace{1cm} a_{12}^* = -\sigma^S f_{kk} > 0$$

$$a_{21}^* = -\gamma^*_\sigma - \gamma^*_\alpha \mu > 0 \hspace{1cm} a_{22}^* = \beta \equiv a_{22} > 0.$$  \hspace{1cm} (A.4)

$E^* = k^* - k^S$ is a scalar, and $B^*$ is a $2 \times 1$ matrix of coefficients also evaluated at an initial stationary equilibrium of zero distortionary taxes

$$b_{11}^* = \sigma^S f_{kk} \equiv -a_{12}^* < 0; \hspace{1cm} b_{21}^* = \alpha^S f_{kk}(\gamma^*_\alpha \mu + \gamma_\alpha)/\lambda_1 < 0$$  \hspace{1cm} (A.5)

The solution for the differential system (A.3) follows the usual methods, see e.g., Braun (1983). The characteristic equation of the homogeneous part of (A.3) has a negative
determinant implying that the roots are real and have opposite sign. This defines a saddle-point. Denote the stable root of the homogenous part by $\lambda_2$ where

$$\lambda_2 = \left\{ \beta - \left[ \beta^2 + 4\sigma^2 f_{kk}(y_\alpha + y_\alpha^*) \right]^{1/2} \right\}/2 < 0,$$

implying that $k_0^*$ is initially predetermined and $\sigma(0)$ free to take initial jumps. Also from the homogeneous part, given the initial condition $k_0^*$, relationships for the constants of determination are appropriately found. Then, differentiating the particular solution to the non-homogeneous part and equating with the differential system (A.3) provides solutions for the two constants of determination for the non-homogeneous part.

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Notes

1. In our framework, we assume that labor is inelastically supplied, this being an important source of difference from previous small open economy models of Sen and Turnovsky (1990), Turnovsky and Sen (1991), and the one-good model of Kim (2001). The analysis here also differs from the incomplete markets assumption of Turnovsky (1997, Chapter 7) in that there is perfect capital market integration, i.e., complete markets for capital ownership domestically and abroad. It also differs from Lee (1995), since in Lee’s paper the assumption that the static elasticity of substitution between the two goods is identical to the intertemporal elasticity of substitution renders that analysis equivalent to the incomplete markets case. Brock and Turnovsky (1993) provide an analysis of the welfare effects of differential tariffs in a small open economy.

2. The details of the technical solutions for this model are available from the author in an extended appendix. The analytical model of Cantor and Mark (1988) also considers logarithmic preferences and Cobb-Douglas technology as a benchmark case.

3. This result was shown in Turnovsky and Bianconi (1992) for the one-good case. The intuition for this remark can be traced to Becker (1980) who showed in a different context that if the discount rates diverged, the ‘country’ with the lowest discount rate would accumulate all the world wealth asymptotically. In the benchmark equilibrium, because $N_0 > 0$, it implies that $\mu > 1$. Note also that $a^*$ is in units of the domestic good, thus in terms of the foreign good $a^*/\sigma$ can be different from $\mu a$. A proof for the two-good case is available upon request.

4. This model presents the hysteresis property that temporary changes have permanent effects as can be seen from expression (15d), a property shared by Sen and Turnovsky (1990) for example. However for analytical tractability I consider here only the case of permanent changes to the equilibrium. The hysteresis property of the model is desirable from an empirical perspective since real exchange rate time series data, mainly in the floating period, do present unit roots, see for example Mussa (1986).

5. Equations (18a,b) are obtained by considering the derivatives of the initial domestic and foreign wealth with $dW_0 = dW_0^* = 0$, $dW_0 = dW_0^*$, and $dW_0 + \sigma^2 dW_0^* = \left( N^2 / \sigma^2 \right) d\sigma(0)$. Then, computing the derivative of the constant share of the domestic country wealth in total wealth, $W_0/(W_0 + \sigma(0)W_0^*)$, and setting it equal to zero gives another relationship between $W_0$, $W_0^*$, and $\sigma$ as $(dW_0/W_0) - (dW_0^*/W_0^*) = d\sigma(0)/\sigma(0)$. An algebraic solution for these three relationships gives (18a) and (18b) directly.

6. The positive initial relationship between $N$ and $\sigma$ is consistent with the observation that, in this model, physical capital does not relocate instantaneously, see e.g., Obstfeld (1989).
7. This result may be contrasted with the small open economy model of Turnovsky and Sen (1991) where an increase in \( g \) financed by a lump sum tax does present transitional dynamics. The difference is that in their model there are adjustment costs to capital accumulation and labor is endogenously supplied, hence this form of government spending has an effect on the long run capital stock since it is the long run capital/labor ratio that is constant in their model. In an extended version of this paper available upon request, I calculate the effects of an increase in government spending financed by a tax on the consumption of the domestic good and a tax (tariff) on the consumption of the foreign good. The results are similar to the lump sum tax case shown here.

8. This is due to the initial wealth effect which is absent in the one-good models of Turnovsky and Bianconi (1992), Bianconi (1995), Bianconi and Turnovsky (1997), and Kim (2001). In the small open economy model of Turnovsky and Sen (1991) this impact effect is analogous except that in their model it is driven by adjustment costs whereas here it is driven by the initial relative price (wealth) effect.

9. This is consistent with the findings of Backus, Kehoe and Kydland (1994) based on calibration procedures and the findings of Froot and Rogoff (1991), Chinn and Johnston (1996) and Chinn (1999, 2000) based on traditional econometric methods. The long run effect on the real exchange rate in this case is different from the small open economy of Turnovsky and Sen (1991) where the long run real exchange rate appreciates. In their model, under their finance method and plausible conditions, government spending has an expansionary long run effect on domestic capital and variable labor, whereas in my model the long run effect on the capital stock is contractionary with fixed labor.

References


FISCAL POLICY AND TERMS OF TRADE


