Twelve-Tone Technique: A Primer

Basic principles. A piece of music composed in twelve-tone technique assumes a preexisting "precompositional," meaning the composer chooses it before composing any actual music with it) ordering of the twelve pitch-classes of the chromatic scale. This ordering is called by various authors the set, or the series, or the row (today chiefly in England) of the piece. All the pitches in the piece are (or should be) referable in some way to the particular set; this is the fundamental premise of twelve-tone technique.

In Schoenberg's technique, there are four basic forms of the set. The original ordering is called the prime (abbreviated P). The other three forms are the inversion (I), the retrograde (R), and the retrograde inversion (RI). Any of the four forms may be transposed to any degree of the chromatic scale, making 48 possible forms in all of any given prime set.

Any pitch-class in the set may be used compositionally in any octave; enharmonic equivalence of each pitch-class is assumed. Thus pitch-class C includes all B sharps and all D double-flats. The choice of one or other enharmonic notation is determined principally by convenience in reading, since in most twelve-tone music there are no tonal considerations (a famous exception: Berg's Violin Concerto).

One often-stated principle of twelve-tone composition is that no pitch-class is to be repeated until all the other eleven members of the set have had their turn. This is an oversimplification. In most twelve-tone works it is a commonplace to find notes repeated any number of times in immediate succession, or sometimes groups of notes repeated successively. (Curious footnote: There are exactly 12!, that is, 12×11×10×9... possible twelve-tone sets. This is a large number, 4×1001600 to be precise. The number of possible prime sets is one-forty-eighth of this number, or 9,979,200, which ought to be enough for anybody.)

In actual compositional use, the set, in whatever form the composer selects (for any compositional reason) at the moment, typically appears in serial order (that is, one note after the other; hence the name "serial composition"), within one voice or distributed between several. Just as typical, however, is the occurrence of simultaneous notes in the set, for polyphonic reasons. In that case, the serial ordering of the pitches sounding simultaneously has no relevance at that point.

 Paradigms. Analysis (or for that matter composition, if the composer feels he needs it) of a twelve-tone piece is facilitated by a suitable graphic representation of the 48 set forms. A typical arrangement is a page of twelve-page music paper with the prime form at the left and the inversion form at the right, beginning at the top with the untransposed prime (P) and the untransposed inversion (I), then the first transposition (P, I) on the next staff, and so forth down the page. On page 3, below, is shown the set for Schoenberg's Fourth String Quartet written out in just this way. R and RI forms can be read backwards on this chart, or a separate page for them, with R and RI reading left to right, can be made. The consecutive arrangement has the advantage that one can instantly locate the desired form simply by reading the index number in the margin, or by counting. (Clever Schoenberg, who loved tinkering, took twelve identical vertical strips of music paper, each with twelve half-inch-wide staves in a column, and wrote the chromatic scale on each one; then he pasted the ends of each together until he had
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twelve little chromatic-scale hoops, which he looped over a cardboard cylinder. By tuning each hoop until he had the notes of the desired set in a horizontal line, he could make all the other transpositions fall into place automatically.

Numerical designations. For those not intimidated by a little elementary arithmetic, a numerical system of notation can be very helpful in twelve-tone manipulation. Milton Babbitt's numerical conventions are practical and of wide use in America. The first convention is that the twelve pitches of every set are numbered in order beginning with zero. (In England, rows are generally numbered beginning with 1.) Additionally, the pitches are measured by their semitonal intervallic distance above (not below) the initial pitch, also beginning with zero. (So-called "normal form" places the initial pitch as the lowest, with all the other pitch-classes within the octave above it.) Accordingly every member of the set can be designated by a number couple, consisting of the order number followed by the pitch number. The set from Schoenberg's Fourth Quartet can thus be represented in its untransposed prime form (P) as follows:

0,0 1,11 2,7 3,8 4,3 5,1 6,2 7,0 8,6 9,5 10,4 11,9

and its untransposed inversion (I):

0,0 1,1 2,5 3,4 4,9 5,11 6,10 7,2 8,6 9,7 10,8 11,3

(The chart on page 3 shows all the P and I forms in ascending chromatic order; the order numbers are the boldface numerals at the top, zero through 11; the pitch-class numbers are the smaller numerals below the individual notes.)
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Prime forms

Inversion forms

Prime and Inversion forms read left to right; order numbers are indicated by the large boldface numerals above, from zero through 11. Retrograde and Retrograde Inversion forms are read right to left, starting respectively at the last note of the Prime and Inversion forms. (Order numbers for 8 and 9 forms are not given.)
The graphic representation on page 3 has the disadvantage that all the 48 forms take up a lot of space. Another representation, more economical of space but requiring some experience to use comfortably, is the matrix representation devised by Milton Babbitt, in which the transposed primes are arranged not in vertical chromatic order but in the order in which their initial pitches appear in the untransposed inversion. With this arrangement, primes are read horizontally, left to right, and inversions vertically, top to bottom; one has to be aware of which transposition is involved in each case, because the transposition number does not usually correspond to the numerical position on the chart. (See the same Schoenberg set written out in this way, page 5, below.) From this it is easy to write a matrix which is made up of these couples instead of pitch-letters.

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It is even easier to write a matrix which contains only the pitch numbers, the order numbering being achieved by counting, just like pinpointing locations on a grid map. Some analysts, even composers, find this method particularly convenient to use.

\[ \begin{array}{cccccccccc}
0 & 1 & 1 & 12 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\end{array} \]

The order-number-pitch-number couples are useful for obtaining any inversion or any transposition from a given prime. The pitch numbers of the inversion are found by subtracting the corresponding pitch numbers of the prime from 12. (Because I have a poor head for subtraction, I confess that I always do this operation by conventional notation, from note to note.)
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Transposition is accomplished in the same way, by adding the number representing the transpositional interval to each of the pitch numbers in succession, modulo 12 (that is, 12 is subtracted from interval numbers 12 or higher).

If both operations are to be carried out, it is important to do the inversion first, and the transposition afterward, or problems of noncommutativity may result.

Twelve-tone technique in operation. Schoenberg and others presumed, theoretically speaking, an equivalence of all 48 forms of any particular set. For compositional purposes, however, a composer inevitably is selective. Why the composer chooses, for any given moment of music, any particular set-form or forms is a matter of composition; the system does not require, for example, that all 48 forms be used. It is safe to say that there are certain things that are avoided. One does not find in Schoenberg’s twelve-tone works, for example, a melody written with collateral first-inversion triads (à la faubourdon), such as could be achieved by using P₁ in an upper part and simultaneously P₃ and P₅ in two parts below it. This would be a matter of style, perhaps, but it is also in keeping with Schoenberg’s general principle that twelve-tone technique is a technique of atonal music, and parallel triads are, after all, a hallmark of tonal music. On the other hand, it is just as obviously likely that the composer chooses to use one or another different set-forms at a given moment precisely because certain pitches, or certain combinations of pitches, are what are called for. The 48 forms allow a great deal of flexibility for this. In the Walzer from Schoenberg’s Five Piano Pieces, Opus 23, said to be his first completed twelve-tone work, only one form of the set is used, and by definition that would be the untransposed prime (P₁). (See Piston-DeVoto, Harmony, fifth edition, Example 12-11.) But from the mature twelve-tone standpoint one would have to regard that as a primitive use of the technique—which, Schoenberg would be the first to argue, says nothing whatever about the quality, primitive or not, of the music itself.

Schoenberg was justifiably proud of his technique (”Today I have discovered something that will assure the supremacy of German music for a thousand years” — words which in the light of subsequent history, musical and otherwise, sound uncomfortable to us today). Perhaps he treasured it most of all for the great flexibility which it permitted in composition while at the same time guaranteeing a philosophical unity of any particular work composed with a single set (”It does not seem right to me to use more than one row [in a given composition].”) Thus it is all the more remarkable that Alban Berg, his devoted and beloved disciple, differed fundamentally from Schoenberg’s stated principles in his own very first works using the technique. There are two aspects to keep particularly in mind in Berg’s twelve-tone practice in the Lyric Suite. The first is that Berg never employs retrograde or retrograde-inversion forms in any of his twelve-tone works, unless, and only unless, the entire substance of the music itself is retrograde (examples in Der Wein, acht, the Chamber Concerto [which is not a twelve-tone work], and of course the Lyric Suite itself). The second is that Berg did not inevitably wish to be limited to the specific intervallic configuration of a single set, and accordingly the first movement of the Lyric Suite uses not one but three different sets. These three are all related combinatorially (see below), but they are not interconvertible under any of the standard Schoenberg operations.
Why would Berg restrict himself invariably to the use only of P and I forms? He did not offer any explanation of this that we know of. But it is probably fair to say that for Berg, much more than for Schoenberg or Webern, the twelve-tone set represented first of all a kind of theme, often in a specifically melodic manifestation. As such, the twelve-tone theme would normally be established at some kind of primary pitch level, and would appear in transpositions only at particularly important places or for determinable reasons. Similarly, a twelve-tone theme when inverted retains much of its original and essential melodic character, including the identical rhythm and a comparable, even though mirror-reflected, melodic shape. These properties do not usually obtain, however, when the melody is retrograded, in which case the rhythm and contour are normally completely different from the original, and the retrograde relationship is not normally perceived by the ear. We have centuries of tradition to fall back on in the matter of bearing melodies upside-down, from Bach to Brahms (remember "How lovely is thy dwelling place") and later, but we cannot hear time-reversal, and that is why there are so few examples of melodic retrogrades in history, compared with melodic inversions.

Combinatoriality. This property, which was much exploited by Schoenberg, is best illustrated by an example. Here are the P₃ and I₉ sets of Schoenberg’s Fourth Quartet:

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A

B

secondary set (boxed)
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We have divided each set into two hexachords, A and B. It will be noticed that the A hexachord of B, and the B hexachord of I₉ are identical as to content, that is, they contain the same six pitch-classes even though not in the same order. The six pitches of the A hexachord of I₉ therefore, will be completely different from the A hexachord of P₃. The practical result is that the composer may state the A hexachord of P₃, and follow it with the A hexachord of I₉, yielding twelve different tones. (Such a succession of hexachords is called a secondary set, a simultaneous combination of them, an aggregate.) It follows that the A hexachord of P₃ would combine thus with the A hexachord of I₉, or P₃ with I₉, etc. A set showing this property is said to possess hexachordal inversional combinatoriality at interval 3.

A set which possesses a combinatorial form at at least one transposition of P, I, R, and RI is called all-combinatorial. If a set has a combinatorial form at some transposition of P or I or R or RI but not all four, it is called semi-combinatorial. Every set is at the very least semi-combinatorial by definition, if only because, in the most trivial instance, the A hexachord of P₃ and the A hexachord of P₉ form a twelve-tone secondary set.

Every all-combinatorial set, when arranged in normal form and transposed to a standard degree (by custom, beginning on middle C) will belong to one or another class of six all-combinatorial source hexachords, shown here:
All three of the sets in the first movement of Berg's *Lyric Suite* are related in this way, being different versions of the "all-combinatorial major hexachord." To put it another way, the three sets are *all-associative* with respect to each other.

**Special sets.** Schoenberg, Berg, and Webern were all intensely interested in the special properties of particular twelve-tone sets. In the case of Schoenberg and Berg, it is probably fair to say that these properties interested them chiefly from the theoretical standpoint, because the special properties are not notably exploited in the music in which they used these sets. Webern, on the other hand, seems to have made a particular effort to realize in his actual music the special characteristics of his sets (this is, however, as area in which I am pretty vague and I won't say much about it).

A good example of what I mean by "special set" is the *all-interval* set used by Berg as the principal set of the three in the first movement of the *Lyric Suite*. This set, discovered by Berg's pupil F. H. Klein, displays all eleven possible intervals in its adjacent pitches (semitone, minor sixth, minor third, minor seventh, etc.). (Berg thought it was the only all-interval set, but in fact 1,928 all-interval sets are possible.) There is no evidence that I can see in the *Lyric Suite*, however, nor any reported by others that I know of, that Berg actually exploited this all-interval property in any coherent way; apparently he used the set as he might have used any other set. There are, however, other properties of this unusual set which do appear to be reflected in the music itself. One is that alternate notes in the hexachords form cycles of ascending fifths or fourths; this property is related to the second set of the three, the cycle-of-fourths set first appearing in the cello in measures 7-9. Another property is degeneracy (see below).

Webern was especially interested in sets which begin with a pattern of two or three intervals which are then, in one or another permutations, repeated in the subsequent *unfolding* of the set. Such sets are called *derived sets*; a good example is the set of his Concerto for three instruments, Opus 24. If the first trichord is designated \( p_0 \), the trichords \( p_1 \), \( p_2 \), and \( p_3 \), generate the remainder of the set.
The set of Webern's String Quartet, Opus 28, begins with the famous tetrachord B-A-C-H. The second tetrachord is a transposition of the RI of the first tetrachord; the third is a simple transposition of the first tetrachord. Can you figure out what the second and third tetrachords are? (NB, I don’t myself know whether this problem has a unique solution, although I suspect that it does.)

Schoenberg discovered what he called a "Miracle Set" (Wunderreihe), a symmetrical set which is inversionally combinatorial at intervals 3, 5, 7, 9, and 11. (It reduces to the all-combinatorial source hexachord No. 2 in the list.) He liked it enough to register it for copyright in 1950; I am not certain whether he ever used it in a composition, but I do recall a rumor which I have not verified, namely that he had already unwittingly used the set in an early twelve-tone work of his, the Four Pieces for mixed chorus, Op. 27. Someone might care to investigate this. (See Josef Rufer, Arnold Schoenberg: A Catalogue of His Compositions, Writings, & Paintings, 1962, pp. 150-151 and Plate xxv.)

Berg's Violin Concerto (1935) makes use of the most famous of all twelve-tone sets. This set unfolds four overlapping triads, G minor, D major, A minor, and E major, concluding with a segment of a whole-tone scale; this in turn maps with the initial four notes of the chorale melody Es ist genug, which is given by Berg phrase by phrase, a twelve-tone harmonization alternating with Bach's harmonization (Chorale No. 216, from Cantata No. 60). Question: why does Berg begin the initial statement of this chorale with $P_1$, instead of the more fundamental $P_0$? (See Piston-DeVoto, Harmony, fifth edition, Example 32-13.)

Degenerate sets. The all-interval structure of the first Lyric Suite set is symmetrical; the first interval (semitone) is balanced by the last (the complementary interval, major seventh), the second (minor sixth) by the next-to-last (major third), etc. It is not hard to see that this symmetry causes the set to produce its own retrograde in the interval of 6, that is, $P_6$ is actually equal to $S_0$. It follows that $P_1 = P_5$, $P_2 = P_6$, $P_3 = P_8$, and so forth, and that this set will produce only 24 nonequivalent forms instead of 48. Such a set is called a degenerate set. (Edward T. Cone says, "Since, in the minds of many, all twelve-tone rows are degenerate, I suggest, as a less loaded word, 'redundant.'" See "A Budding Grove," Perspectives of New Music III/2, pp. 38-48, spring-summer 1965.)

For further reading. There is a vast literature on twelve-tone music and its theory, including a number of book-length studies. Some good introductory material can be found in the various music dictionaries, perhaps going a little beyond what I have set down here. But the best exposition I know of is still the standard of over forty years, George Perle's Serial Composition and Atonality: An Introduction to the Music of Schoenberg, Berg, and Webern. This book is economically -- some would say too tersely -- written, but is very clear for all that, and most important, it deals with music much more than with the purely theoretical aspects.
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Schoenberg's own essays on twelve-tone technique can be found in Style and Idea: Selected Writings of Arnold Schoenberg, edited by Leonard Stein (1975, London, Faber & Faber, now available in paperback but I forget the American publisher). As so often with his writing, which are very numerous, these essays are more interesting for what they reveal about Schoenberg the man than for any particularly penetrating insights about his theory or even his music.

Serial Music: A Classified Bibliography of Writings on Twelve-Tone and Electronic Music, by Ann Phillips Basart (1965), lists some 823 individual items published up to that year. These include a good cross-section of useful reading, but of course the field has burgeoned in the 41 years since.