Math 150-02  
Homework 10  
Spring 2013

Instructions: You may work together on this, but if you do, YOU MUST tell me with whom you worked, and on which problems. Furthermore, all answers must be written in your own words. This is due back by Tues., 3pm, April 30, 2013. (HINT: EACH OF THESE PROBLEM SOLUTIONS IS ONLY A COUPLE OF LINES LONG!) This HW is OPTIONAL.

1. Let $A = UΣV^T$ be the singular value decomposition for $A ∈ \mathbb{R}^{m×n}$. Show that the eigenvalues of $AA^T$ (NOT $A^TA$) are the squares of the singular values of $A$, and the the eigenvectors of $AA^T$ are the columns of the matrix $U$.

2. Suppose $A$ is symmetric 2x2 but indefinite (meaning its eigenvalues are real but fall on either side of 0). Then $A$ is orthogonally diagonalizable ($A = PDP^T$), but of course the diagonal elements can be negative or positive. From $A = PDP^T$, find an SVD of $A$.

3. Suppose $A$ is square and invertible. Construct a singular value decomposition of $A^{-1}$ from the SVD of $A$.

4. Use the SVD of $A$ to show that $\|A\|_F = \sqrt{trace(A^TA)} = \sqrt{\sum_{i=1}^r σ_i^2}$