Cortical Constraint Method
for Diffuse Optical Tomographic Brain Imaging

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Outline

- DOT Background
- Parametric Models for Optical Properties
- Numerical Results
- Conclusions and Future Work
Diffuse Optical Tomography

- tissue illuminated by near-infrared, frequency modulated light
- light detected in array(s)
- model of physics used to infer optical properties of tissue
- Differences in optical properties $\Rightarrow$ 3D images with hot spots
Forward Model

\[ Af \approx g \]

Difficulties in solving:

- Underdetermined
- Large number of voxels
- Sensitive to noise in data
Inverse Problem

Typical Tikhonov regularization:

$$\min_{f} \|W(Af - g)\|_2^2 + \lambda^2 \Omega(f),$$

$f$ is absorption (diffusion) perturbation image(s).

Difficulties:

- Choosing $\Omega$
- Choosing $\lambda$
- Computational complexity
Assumptions to exploit:

- region of activity is constrained to cortex.
- **1-1 map** from cortical surface to subset of $\mathbb{R}^2$.

**KEY:**
Parameterized, 2D shapes can be mapped to shapes on surface of the cortex.
Inverse Problem Revisited

\[
\min_p \| W (\tilde{A} \tilde{f}(p) - g) \|_2^2
\]

where

- vector \( p \) describes anomaly shape(s) & optical property(ies).
- \( \tilde{f}(p) \) represents the image(s) of absorption/diffusion
- \( \tilde{A} \) is sampled at points on the cortical surface.
Let $h(x, y)$ be the height of the surface of the cortex at $x, y$. Then for $r = (x, y, h(x, y))$:

$$\mu_a(r) = \alpha_a \left(1/2 + 1/2(\tanh(-\beta p(x, y)))\right)$$

$$D(r) = \alpha_d \left(1/2 + 1/2(\tanh(-\beta \tilde{p}(x, y)))\right)$$

The unknowns:

- $\alpha_a, \alpha_d$
- $v_a, v_d$ – vectors of polynomial coefficients.
Discrete Model

If $f_{\mu_a}$ and $f_D$ are the vectors of discrete values $\mu_a(r_i), D(r_i)$,

$$\tilde{f}(p) = [f_{\mu_a}]$$  or  $$\tilde{f}(p) = \begin{bmatrix} f_{\mu_a} \\ f_D \end{bmatrix}$$

and

$$p = [\alpha_a, v_a]$$  or  $$p = [\alpha_a, v_a, \alpha_b, v_b].$$
Numerical Results

\[ \min_p \| W(\tilde{A}\tilde{f}(p) - g) \|_2^2 \]

- Experiments performed in Matlab 6
- Nonlinear least squares solver: Levenberg-Marquardt
- 4th order polynomials
- Stopping: where \( \| W(\tilde{A}\tilde{f}(p) - g) \|_2^2 \approx \| Wn \|_2 \)
- \([-1, 1] \times [-1, 1] \) region, \( 31 \times 31 \) grid
- Noise-to-signal ratio: \( \| Wn \|_2 / \| W g_{true} \|_2 \)
Mockup of Cortex

Mockup of brain surface

SPIE, Aug. 6, 2004 – p.1
Location of Sources/Detectors

Sources = x.  Detectors = o
Starting Guess

starting shape

h(x,y)

SPIE, Aug. 6, 2004 – p.13
Ex. 1: Absorption; $\alpha_a : 30 \, ; \, \text{NSR: 5\%}$
Ex. 1, Cont.

true $\mu_a$

reconstructed $\mu_a$

SPIE, Aug. 6, 2004 – p.15
Ex. 2: $\alpha_a: 30; \alpha_d: 1/750; \text{NSR}: 1\%$
Ex. 2, absorption, projected view

true $\mu_a$

reconstructed $\mu_a$
Ex. 2, Diffusion
Ex. 2, Diffusion, projected view

true D

reconstructed D

SPIE, Aug. 6, 2004 – p.19
Ex. 3, $\alpha_a = 20$; NSR: 10%
Ex. 3, projected view

true $\mu_a$

reconstructed $\mu_a$
Conclusions and Future Work

- Presented model & algorithm for 3D imaging of optical properties of the cortex
- No additional regularization term
- Other shapes
- Unknown background can be modeled
- Multiple objects
- More realistic brain models
- Clinical data
Acknowledgements

This work supported by the National Science Foundation under grants 0208548 and 0139968