Residential Mobility and the Housing Market in a Two-sector Neoclassical Growth Model*

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Abstract
The impact of residential mobility and competitive housing markets on long run growth is examined using a two-sector general equilibrium overlapping-generations model in continuous time. There is an infinity of agents with finite lives who adjust their housing consumption by moving, which is costly. We explore the model’s steady-state properties, first with a free housing market, then under rent control when the market clears through restrictions on the frequency of moves. Rent controls do not just reduce welfare; they may increase the steady-state capital-labor ratio.

Keywords: Housing markets; frictions; mobility; growth

JEL classification: O41; R21; E1

I. Introduction
Residential mobility and Walrasian price setting are central features of housing markets. Our model allows us to consider the impact of rent control, stylised as a constraint on moving, and of moving costs more generally, on long-run growth in general equilibrium. Residential mobility has attracted considerable attention, but has not previously been set in a general equilibrium context.1 It has long been recognized that policy interventions in housing markets can and often do have substantial and typically unintended effects on households’ moving behavior. In particular, renters in rent controlled housing markets move less frequently and consume suboptimal

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quantities of housing. Administrative allocation of publicly owned housing and high transaction costs more generally have similar effects. As shown by Mayo and Stein (1995), low rates of residential mobility demonstrably make labor markets less efficient and, as our paper shows, can affect economic growth as well.

Housing markets in market economies allocate shelter to households; through their role in setting the price of the single most important and commonly held asset in household portfolios, housing equity, they determine wealth in those economies. Residential mobility is an equilibrating factor crucial to this dual allocative function of housing markets. Because of the durable and immobile nature of housing capital, households must usually move if they are to change the quantity of housing services they consume. Because moving is costly, most households, whether they rent or own, enter the market infrequently, and only a small fraction of the units of housing stock is on the market at any point in time.

We use a dynamic two-sector general equilibrium model in the neoclassical tradition to analyze the impact of price setting and mobility in housing markets on long-run growth. Our model introduces residential mobility by having agents optimize with respect to both their consumption (of housing and non-housing goods), and the number of residential moves they make over their lifetimes. Agents’ demand for housing grows during their lifetimes and they can adjust the quantity of housing they consume only by moving. Because moving is costly, individuals adjust their housing consumption at a finite number of points in time during their lives.

After considering general equilibrium in the steady state, we introduce a stylized form of rent control, a ceiling on the price of housing services below the level at Walrasian general equilibrium. Another innovative feature of our model is the allocation of the housing stock under rent control: a less than optimal frequency of moves rations aggregate housing demand, reconciling the housing supply with the ceiling on rents. We then extend the model to the case of a growing population, where new construction requires only capital, or capital and labor under non-increasing returns to scale. We show that in this case, housing market reform in the form of abolition of rent control impacts long-run growth by affecting the long-run equilibrium rate of interest, and thus factor intensities in both sectors.

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3Two-sector models for studying the housing market were used by Feldstein in a series of papers, eg. (1982), on the impact of inflation on investment when tax rules are not indexed.

II. The Behavioral Model

Consumption decisions are made by households which enter the economy and die continuously; each has a finite lifetime, and households differ only with respect to the time they enter the economy.\(^4\) There is an infinite number of overlapping generations of households coexisting at any point in time. Although each household adjusts its housing consumption at a finite number of points in time, with a large number of households this discreteness is smoothed in the aggregate, so that market demand can be examined using standard tools.

A household’s optimization problem in an unrestricted environment is as follows. Consider a household which is formed at time \(t_0\) and dies at time \(t_0 + T\). Let \(\{t_0, t_1, \ldots, t_{n-1}\}, t_0 \leq t_1 \ldots \leq t_{n-1} \leq t_n \equiv t_0 + T\), be the points in time when adjustments in the quantity of housing take place, with the corresponding housing quantities being \(\{H_0(t_0), \ldots, H_{n-1}(t_0)\}\). That is, \(H_j(t_0)\) is the housing stock purchased at time \(t_j\) by a household formed at time \(t_0\), \(j = 0, \ldots, n - 1\). A unit of housing stock generates one unit of housing services per unit of time. Households consume a constant quantity of housing services during each residence spell (the interval between moves); they can adjust the amount of housing services they consume only by moving.\(^5\) Utility cost \(Bn\) is the only cost associated with the \(n\) moves the household plans to make over its lifetime. Mobility cost parameter \(B\) reflects how easily vacant homes can be found, a function of the economy’s informational structure. A small value of \(B\) is associated with market features which facilitate matching between buyers and sellers or landlords and tenants, such as the role of the real estate brokerage industry.

We examine the economy along a steady state. The rate of interest \(I\) is therefore equal to its steady-state value; cf. Cass and Yaari (1967). Let \(W_0(t_0)\) denote lifetime human wealth for a household formed at time \(t_0\), and \(\chi\) the rate of housing appreciation. Let \(P_C(t), P_H(t)\), and \(Z(t)\) denote, respectively, the nominal prices of non-housing consumption and of a unit of housing stock, and the rental rate of a unit of housing stock at time \(t\). Arbitrage requires that \(Z(t) = (1 - \chi)P_H(t)\), where we have used the claim, which is verified below,

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\(^4\) The behavioral model we use here builds on those of Amundsen (1985) and Englund (1985), as well as on Romer’s (1986) model of the transactions demand for money. It is described in much greater detail in Hardman and Ioannides (1995).

\(^5\) This feature reflects the quasi-fixed nature of housing. Housing deteriorates slowly. If housing consumption were allowed to vary exogenously during a residence spell in our model, the principal effect would be a reduction in analytical tractability. The substance of our model applies as long as mobility costs have a fixed component.

that \( P_H(t) = P_H(t_0)e^{\theta(t-t_0)} \). \( C(t) \) denotes non-housing consumption during the residence spell which begins at time \( t_j \).

The lifetime budget constraint is written as

\[
W_0(t_0) = \sum_{j=1}^{n} e^{-I(t_{j-1}-t_0)} \int_{t_{j-1}}^{t_j} e^{-I(t-t_{j-1})} P_C(t) C(t) \, dt + \sum_{j=1}^{n} H_{j-1}(t_0) \int_{t_{j-1}}^{t_j} Z(t) e^{-I(t-t_0)} \, dt.
\]

(1)

We assume that households are renters. The housing stock is owned collectively by the population, and claims to ownership are continuously passed on by agents who die to those who are newly born.

Households value housing and non-housing consumption during their own lifetime and leave no bequests other than housing. A household’s lifetime utility maximization problem is stated as follows. A household chooses \( \{ C(t); H_0(t_0), \ldots, H_{n-1}(t_0); t_1, \ldots, t_{n-1} \} \) so as to maximize

\[
\Omega_0 = \sum_{j=1}^{n} \left\{ \alpha_C \int_{t_{j-1}}^{t_j} \ln C(t) \, dt + \alpha_H(t_j - t_{j-1}) \ln H_{j-1} \right\} - Bn,
\]

(2)

where parameters \( \alpha_C \) and \( \alpha_H \) satisfy \( \alpha_C > 0, \alpha_H > 0 \), and \( \alpha_C + \alpha_H = 1 \), subject to budget constraint (1).

Hardman and Ioannides (1995) show that a household’s total expenditure on housing is a constant fraction of its initial wealth, \( \overline{H} = \alpha_H W_0 \), and that the rate of consumption is \( C(t) = (\alpha_C W_0/T P_C(t_0)) e^{I(t-t_0)} \), \( t_0 \leq t \leq t_0 + T \). At the optimum, moves occur at \( N \) equidistant points in time. The demand for housing stock at the time of the \( j \)th move, by a household which enters the economy at time \( t_0 \), is

\[
H_j(t_0) = \frac{1}{N} \alpha_H W_0(t_0) \frac{1}{P_H(t_0)e^{-(I-\chi)(jT/N)}[1 - e^{-(I-\chi)(T/N)}]},
\]

(3)

\[ j = 0, \ldots, n - 1. \]
Our assumption of no time discounting in our model (also made by Amundsen (1985) and by Romer (1986) in order to simplify an otherwise intractable optimization problem) is principally responsible for the result that housing demand grows over a household’s lifetime. This conflicts with both the standard life cycle model and empirical findings of researchers who use the residence spells model and who find that housing demand peaks in midlife.\(^8\) Recent research on the housing decisions of the elderly has shown that elderly households often do not choose to reduce their housing consumption, much less than the strict lifestyle model predicts.\(^9\)

**Optimal Times of Moves**

Hardman and Ioannides (1995) show that the optimal number of moves, considered for convenience as a real-valued variable, \(N\) is obtained implicitly in terms of the unique root \(x^*(\cdot)\) of the transcendental equation

\[
((I - \chi)(B/\alpha_H) + x - \frac{1}{2}x^2)/(((I - \chi)(B/\alpha_H) + x - \frac{1}{2}x^2) = e^x. \text{ Given } x^*(\cdot), N(I - \chi; B/\alpha_H) \equiv (I - \chi)[T/x^*(I - \chi(B/\alpha_H))]. \text{ In the same paper, comparative statics for } N^*(\cdot) \text{ confirm the intuition that the greater the disutility of each move, the smaller the optimal number of moves in a lifetime. The number of moves in a lifetime tends to } \infty (0) \text{ when the utility cost per move tends to } 0 (\infty). \text{ The impact of a change in } I - \chi \text{ is ambiguous. Households oversize at the beginning of each spell and undersize at the end of the spell, and the gap from the continuous adjustment case is increasing with } B. \text{ This result is indeed consistent with empirical findings by Henderson and Ioannides (1989) and Edin and Englund (1991).}

**III. General Equilibrium**

We now set our model in an infinite overlapping-generations general equilibrium framework along the lines of Cass and Yaari (1967) and Romer (1986), but augmented to allow for two sectors. Individuals supply their labor inelastically and their incomes consist of both labor earnings and asset income. Individuals borrow and lend freely in perfect capital markets. Wealth at birth consists of human capital only, that is, the discounted value of the stream of lifetime income receipts. Expressions for aggregate housing and non-housing consumption are easily obtained. After examining the case of no population growth, we consider the impact of population growth, which complicates the aggregation problem and may give rise to capital gains to housing.

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\(^8\)See Rosenthal (1988) and Henderson and Ioannides (1989).

Steady-state Equilibrium with No Population Growth

We consider first the case of no population growth and a given (constant) stock of housing. The housing stock is owned privately. A fraction of it is always “on the market”, changing hands as new individuals are being born. Old individuals die when they reach age $T$. Individuals adjust their housing consumption according to their optimal plans. The housing stock is assumed not to depreciate and the real price of housing stock is constant over time, $P_H(t) = P_H$, at the steady-state equilibrium, $\chi = 0$. This implies that the equilibrium rent is given by $Z(t) = IP_H$.

The assumption of an infinite number of overlapping generations in the Cass-Yaari-Romer model means that a new generation is born at each instant in time. With no population growth, generations are of equal size and their members have identical preferences. Individuals differ only with respect to when they were born. Thus, the inherent discreteness at the individual level is completely smoothed out in the aggregate. We normalize the size of each generation to be equal to $1/T$ so that aggregate population is equal to $(1/T)T = 1$. The non-housing good is assumed to be the numeraire $P_C(t) = 1$. The steady-state general equilibrium is characterized by the rate of interest and the price of housing stock ($I, P_H$).

In steady-state equilibrium the aggregate demand for consumption is equal to the average consumption over an individual’s lifetime. This and (6) yield $C_D = (1/T)\int_0^T C(t) \, dt = (1/T)\alpha_C W_0(e^{IT} - 1/IT)$.

The non-housing good is produced by means of a linear homogeneous production function, which is increasing and concave in both of its arguments. We write this production function in the standard fashion as $L_C f(K_C/L_C)$, where $f' > 0$, and $f'' < 0$, and $L_C$ and $K_C$ are, respectively, the amounts of labor and capital employed in the non-housing good sector. We characterize equilibrium in the non-housing good sector by setting aggregate demand equal to aggregate supply, which is equal to aggregate output minus depreciation of the capital invested in the production of the consumption good, $L_C f(K_C/L_C) = K_C$ (non-housing capital depreciates fully). There is no population growth and all labor is inelastically supplied, $L_C = L$.

Lifetime human wealth, $W_0$, is equal to the present value of the stream of lifetime income, which consists of earnings, $f(k_C) - k_C f'(k_C)$, and of an individual’s share of aggregate housing rental income, $IP_H H_S$, discounted by the equilibrium real rate of interest. We anticipate the fact that at the steady-state equilibrium $k_C$ is a function of the rate of interest $I$ only and write $W_0 = W_0(I, P_H)$:

$$W_0(I, P_H) = \int_0^T [f(k_C) - k_C f'(k_C) + IP_H H_S]e^{-lt} \, dt, \quad (4)$$
where \( k_C \) denotes the steady-state capital-labor ratio, \( k_C = \frac{K_C}{L_C} \), and \( \bar{H}_S \) per capita housing stock. General equilibrium requires that all four markets be at equilibrium, that is, the market for the non-housing good, the housing market, the market for capital, and the market for labor. By using the non-housing good as the numeraire, the three prices, namely \( P_H \), the wage rate and the rate of interest are left to be determined at equilibrium. We have already expressed the wage rate, which is equal to \( f(k_C) - k_C f'(k_C) \), as a function of the real rate of interest (by making use of the factor-price frontier). The demand for capital is determined from the conditions for profit maximization in the non-housing good sector, \( I = f'(k_C) - 1 \), which once inverted becomes:

\[
k_C = k_C(I).
\]

Aggregate consumption demand can also be written as a function of \( I \) and \( P_H \), \( C_D = C_D(I, P_H) \), only.

Aggregate housing demand at time \( v \), which is equal to average demand over an individual’s lifetime is given by

\[
H_D(v) = \frac{1}{N} \sum_{k=0}^{N-1} H_k(v),
\]

where \( H_k(v) \) denotes the housing demand at the beginning of the \( k \)th spell by an individual of generation \( v \) (that is, who has entered the economy at time \( v \)) along the steady state. From (3) and (6) at the steady state with \( \chi = 0 \), and under the assumption that each individual moves \( N \) times during his lifetime, we have that \( H_k \) is independent of \( v \), the time the individual entered the economy, and depends only on the order of the spell:

\[H_k = \frac{1}{N} \frac{a_H W_0(I, P_H)}{P_H} \frac{1}{e^{-I(T/N)k} [1 - e^{-I(T/N)}]} .\]

Aggregate housing demand, when individuals make \( N \) equidistant moves, now follows:

\[
H_D(I, P_H, N) = \frac{a_H W_0(I, P_H)}{P_H} (e^{IT} - 1) \frac{e^{I(T/N)}}{N^2(e^{I(T/N)} - 1)^2} .
\]

If \( N = \infty \), the optimal number of moves, then \( H_D \) gives the unconstrained (“Walrasian”) aggregate housing demand function. \( H_D(I, P_H, N) \) is an increasing function of \( N \) and is bounded upwards by the respective quantity for the frictionless case.
Housing Market Equilibrium

We simplify the remaining exposition by recognizing that the condition for housing market equilibrium can be solved to yield an expression for \( P_H \). The condition for housing market equilibrium with fixed housing supply \( H_S \) is:

\[ H_D(I, P, N) = H_S. \]

We define the auxiliary variable:

\[ \Phi(I, N) \equiv (e^{IT} - 1)(IT)^2 e^{IT} / N^2 (e^{IT/N} - 1)^2. \]

Hardman and Ioannides (1995) establish that \( \Phi(I, N) \) is increasing in both \( N \) and \( I \). Solving for \( P_H \) yields

\[
P_H(I, N) = \frac{1}{H_S} \frac{f(k_C(I)) - k_C(I) f'(k_C(I))}{IT} \frac{\alpha_H(e^{IT} - 1) \Phi(I, N)}{1 - \alpha_H(e^{IT} - 1) \Phi(I, N)}.
\]  

(8)

Equilibrium in the Output Market

The supply of capital is equal to the aggregate savings in the economy, which are in the form of life cycle savings by all individuals, and is equal to aggregate income, minus aggregate spending on consumption. Equivalently, if we work with the market for output, we can write the equilibrium condition in terms of the rate of interest:

\[
\frac{1}{T} \alpha_C W_0(I, P_H) \frac{e^{IT} - 1}{IT} = f(k_C(I)) - k_C(I).
\]  

(9)

General Equilibrium

The general equilibrium price system \( (I^*, P_H^*) \) is fully characterized by the solution to the system of equations (8) and (9) in terms of the price of housing \( P_H \) and the real rate of interest \( I \). We say an *unrestricted* equilibrium obtains if \( N \) in (8) and (9) assumes the value \( N = \mathcal{N} \), the individual-optimizing value.

By using (8) we obtain an expression for housing wealth which we use in (4), the definition of \( W_0 \). It follows that lifetime wealth as a function of \( (i, N) \), \( W_0(I, N) \), is given by
\[ \bar{W}_0(I, N) = [1 - \alpha_H(e^{HT} - 1)^2 \Phi(I, N)]^{-1} [f(k_C(I)) - k_C(I)f'(k_C(I))] \frac{1 - e^{-IT}}{I}. \]

We now use (10) to rewrite (9) and obtain

\[ (1 - \alpha_H)[1 - \alpha_H(e^{IT} - 1)\Phi(I, N)]^{-1} \Theta(IT) = \frac{f(k_C(I)) - k_C(I)}{f(k_C(I)) - k_C(I)f'(k_C(I))}, \]

where

\[ \Theta(IT) \equiv \frac{e^{IT} - 1}{IT} - \frac{1 - e^{-IT}}{IT} = 2 \left( \frac{1}{2!} + \frac{1}{4!} (IT)^2 + \frac{1}{6!} (IT)^4 + \ldots \right). \]

Therefore: \( \Theta(0) = 0; \Theta'(0) = 0; \Theta'' > 0; \Theta''' > 0 \). Eq. (11) is closely related to Cass and Yaari (1967, pp. 239–242) for \( \alpha_H = 0, \alpha_C = 1 \); they show that in general there may be no, one, or several steady-state equilibria.\(^{10} \)

We can characterize the roots of (11) from first principles by considering first the counterpart here of the Cass-Yaari case. The RHS of (11) may be an increasing or decreasing function of \( I \). For \( f(k_C) \equiv k_C^\beta, 0 < \beta < 1 \), in particular, in view of (5) the RHS of (11) becomes \( 1/(1 - \beta) [1 - \beta/(1 + I)] \); it is increasing and concave in \( I \); at \( I = 0 \) its value and its slope are both equal to 1. Therefore a unique nonzero steady state exists; see Figure 1, case (a). Turning now to our case, costly mobility and a housing sector make their presence felt when \( \alpha_H > 0 \) via the term \( \alpha_C[1 - \alpha_H(e^{IT} - 1)\Phi(I, N)]^{-1} \) in the LHS of (11). Since \( \Phi(I, N) \) is increasing in both \( I \) and \( N \), the LHS of (11) is still increasing in \( I \), if \( N \) is exogenous. However, as the properties of \( \mathcal{M} \) as a function of \( I \) are ambiguous, we cannot determine fully the impact of the presence of housing and costly mobility. Unfortunately, even though the presence of costly residential mobility affects only the LHS of (11), its full effects depend on parameter values. In the case of uniqueness, and provided that \( (e^{IT} - 1)\Phi(I, N) > 1 \), the presence of mobility costs will increase the magnitude of the LHS of (11) (which is larger, the larger is \( \alpha_H \)) and thus decrease the equilibrium rate of interest and increase the steady-state capital-labor ratio; see Figure 1, case (b).

\(^{10}\)It is a generic property of overlapping-generations models that none of these equilibria need be efficient; cf. Diamond (1965).
Restrictions in the Housing Market

We define a restricted housing market as one where there is non-price rationing, that is, \( N \) is restricted from attaining \( N^* \) and must be treated as exogenous in (8) and (9). Such restrictions on residential moves are a typical consequence of the operation of rent control or a substantial stock of publicly owned housing. We fully characterize general equilibrium in the restricted case by first articulating a relationship obtained by eliminating \( N \) between (8) and (11). That is,

\[
P_H = \frac{1}{H_S} \frac{f(k_C(I)) - k_C(I)}{\alpha_H \Theta(I) I} - \frac{f(k_C(I)) - k_C(I)f'(k_C(I))}{I}. \tag{12}
\]

Equation (12) underscores the options made possible by the presence of residential mobility in our general equilibrium model: there exists an infinity of values for \((P_H, I)\), which are in effect, parameterized by \( N \). If \( N = N^* \) then solving (8) and (11) implies selecting specific points \((I, P_H)\) on the curve represented by (12).

A typical shape for the RHS of (12) is depicted in Figure 2. The first term in the RHS has the shape of \( f(k_C(I)) - k_C(I) \), except more elongated.
because the term in its denominator is an increasing convex function of $I$. The second term is a monotonically decreasing function of $I$. Requiring that the rate of interest be positive restricts us to segment $PI$ of the curve.

Let us for simplicity assume that parameters are such that the Walrasian equilibrium is unique and denoted by $(I^*, P^*_H)$. Suppose that housing rent $Z^c$ is set at less than $Z^*$, the unrestricted Walrasian value $Z^* = I^* P^*_H$. This implies that $P^c_H < P^*_H$ and non-price rationing in the housing market requires that the number of moves $N$ must be chosen consistently. Since the aggregate demand for housing stock is an increasing function of $N$, (8) can be solved uniquely in terms of $N$ for given $P_H$. The restricted equilibrium value of $N$ is less than $\mathcal{N}(I^* - \chi; B/\alpha_H)$, the unrestricted Walrasian value. When the housing price is restricted below its Walrasian level, the housing market will clear only if residential moves are less frequent than $\mathcal{N}$. A *deus ex machina* or housing administrator\(^{11}\) is,

\(^{11}\)We do not model such an administration. Weibull (1983) offers a formal model of the housing market with trade frictions but does not explore their dynamic general equilibrium implications.
however, needed to enforce the appropriate number of residential moves in the constrained equilibrium.\footnote{Alternatively, agents’ reliance on the frequency of moves as a rationing device could be modeled explicitly, restricting attention to steady states of the economy, so that only equidistant moves need to be considered. We do not explore this approach further here.}

The basic intuition of this outcome follows from the behavioral model. Utility is concave and if one can only move occasionally, one oversizes relative to the price structure at the time of a move. This makes housing at the beginning of a spell a little less valuable than it would be if consumption could be adjusted continuously, driving down the relative value and relative price of housing services as the length of time until readjustment increases.

The presence of restrictions has consequences for general equilibrium. A decrease in $N$ causes a decrease in the LHS of (11). If parameter values are such that there exists a unique steady-state value of the capital-labor ratio, then rent control, combined with the value of $N$ below the Walrasian one which rations the available supply of housing, will decrease the equilibrium rate of interest and thus increase the capital-labor ratio at the steady state.

The policy implications of housing market liberalization depend on parameter values and on the number of equilibria. Liberalizing the housing market will cause welfare at the steady state to improve, for a given constant capital-labor ratio. However, the latter will be affected through the impact of liberalization on consumption demand in (9). We expect that in the case of unique equilibrium, such a change will increase the equilibrium value of the rate of interest and thus decrease the steady-state value of the capital-labor ratio.

Policy interventions like rent controls are obviously inefficient in a static economy. Our analysis further indicates that whereas some proponents sought rent controls in order to ensure lower housing costs, they will instead cause increases in an economy’s non-housing capital. Full dynamic analysis requires a fresh treatment of dynamic efficiency in an overlapping-generations model of the type invoked in this paper. Such a treatment would need to incorporate the features which differentiate our model from the classic treatment in Diamond (1965): more than one good per period, quasi-fixed housing consumption, and the presence of moving costs. We conjecture that when mobility costs are small, removal of rent control will have a beneficial effect when the economy is dynamically inefficient. When the economy is dynamically efficient, removal of rent control causes a decrease in the capital-labor ratio and thus poses a second-best problem.

IV. General Equilibrium with Population Growth

General equilibrium is not interesting unless new housing is produced with capital and possibly labor, as well, under conditions of non-increasing
returns to scale; cf. Weiss (1978). This reflects the presence of one factor (land) in inelastic supply. Consequently, the sector producing new housing earns positive profits in general equilibrium. We assume that all existing housing as well as the sector producing new housing are equally owned by the population and thus their profits are distributed in a lump-sum fashion equally among the population. The two-sector model with population growth follows the classic treatment of Uzawa (1963) and Galor (1992).

With decreasing returns in housing production, the price of housing stock grows exponentially. Our model remains tractable because housing wealth per capita remains finite at the steady state.

**Population Growth and Age Composition**

We model a growing population by retaining the assumption of an overlapping-generations structure and by assuming that the number of households formed per unit of time, \((1/T)e^{\eta t}\), is increasing over time at a constant growth rate \(\eta\). It follows that the age composition of the population is not uniform. We proceed by first characterizing the age composition of the population, implied by our assumptions, which we need in order to derive the aggregate demand for housing.

The total population at time \(t\) is given by \(L(t) = \tilde{L}e^{\eta t}\), where \(\tilde{L} \equiv ((1 - e^{-\eta T})/\eta T)\). The implied distribution of households by age, \(0 \leq v \leq T\), at time \(t\), is given by \(S(v; t) = ((1 - e^{-\eta v})/\eta T)e^{\eta t}\). It is most convenient to work with distributions in terms of generations, where generation \(v\) is the set of households formed during \((v, v + dv)\), \(t - T \leq v \leq t\); generation \(v\) has age \(t - v\) at time \(t\). The normalized density and distribution functions with respect to generations are, respectively:

\[
a(v; t) = \eta e^{-\eta(t-v)}(1 - e^{-\eta T})^{-1}, \quad t - T \leq v \leq t; \quad (13)
\]

\[
A(v; t) = (e^{-\eta(t-v)} - e^{-\eta T})(1 - e^{-\eta T})^{-1}, \quad t - T \leq v \leq t. \quad (14)
\]

**Aggregate Housing Demand with a Growing Population**

The computation of the aggregate demand for housing involves the age composition of the population. Specifically, at time \(t\) the members of generations \((v, v + dv)\), where \(v\) satisfies

\[13\]This implies, in turn, the following normalized density and distribution functions of households in the economy by age, \(0 \leq \tau \leq T\): \(a(\tau; t) = \eta e^{-\eta\tau}(1 - e^{-\eta T})^{-1}; \ A(\tau; t) = (1 - e^{-\eta T})(1 - e^{-\eta T})^{-1}\).
are currently at their \( j \)th spell. Each member of such a generation has a demand for housing equal to \( H_k(v) \), given by (3) for \( t_0 = v \), and moves occur at equidistant points in time. There exists a total of \( (1/T) e^{\eta v} \, dv \) households in generation \((v, v + dv)\). Aggregating their demands requires that we account for the time they enter their \( j \)th spells. Aggregating over all spells yields

\[
H_D(t) = \frac{1}{T} \sum_{j=0}^{N-1} \int_{t - (j+1)T/N}^{t - jT/N} H_j(v) e^{\eta v} \, dv.
\]

In view of (3) and (6), the expression for housing demand during each spell, the above becomes:

\[
H_D(t) = \frac{\alpha_H \tilde{W}_0}{P_H(0) NT} \left( \frac{1 - e^{-(\eta - \chi)T/N}}{1 - e^{-(\eta - \chi)T/N} e^{(I-\eta)T/N - 1}} \right) \left( \frac{1 - e^{-(\eta - \chi)T/N}}{1 - e^{-(\eta - \chi)T/N} e^{(I-\eta)T/N - 1}} \right),
\]

where household lifetime wealth \( \tilde{W}_0 \) must be suitably redefined. Note from (15) that the per capita lifetime wealth decreases at the rate at which the price of housing increases. This, in turn, implies that aggregate housing wealth per capita at equilibrium is independent of time: \( W_H = P_H(0)e^{\chi t} H_D(t) = \alpha_H \tilde{W}_0 \Psi(I, \chi, N) \), where the auxiliary function \( \Psi(I, \chi, N) \), is defined as:

\[
\Psi(I, \chi, N) = \frac{1}{LNT} \left( \frac{1 - e^{-(\eta - \chi)T/N}}{1 - e^{-(\eta - \chi)T/N} e^{(I-\eta)T/N - 1}} \right) \left( \frac{1 - e^{-(\eta - \chi)T/N}}{1 - e^{-(\eta - \chi)T/N} e^{(I-\eta)T/N - 1}} \right) \Psi(I, 0, N) = \Phi(I, N). \]

It is straightforward to show that the RHS of (15) reduces to that of (7) if there is no population growth; \( \eta = 0 \) which, as will be shown implies that \( \chi = 0 \), and \( \Psi(I, 0, N) = \Phi(I, N) \). Aggregation and costly mobility are reflected in a more complicated term in the closed-form expression for housing demand.

**Capital Gains in Housing**

We elaborate on the relationship of capital gains in housing in a growing economy in terms of fundamentals. With non-housing consumption as numeraire, \( P_C(t) = 1 \), and \( P_H(t) \) growing at a rate \( \chi \), \( P_H(t) = P_H(0)e^{\chi t} \). The presence of housing production, under non-increasing returns to scale, and
of population growth is entirely responsible for capital gains in this model and determine $\chi$.

Let production of new housing per unit of time be given by $G(K_H) \equiv K_H^\rho$, $0 < \rho \leq 1$, where $K_H$ is capital used in housing production. We assume as before that both kinds of capital depreciate fully. Capital is allocated across the two sectors to equalize the values of marginal product of capital. Given the equilibrium rate of interest $I$, the capital-labor ratios in the sectors producing housing and consumption goods, $k_H$ and $k_C$, respectively, are determined from $\rho \Gamma P_H(0)e^{\rho I} K_H^{\rho-1} = 1 + I = f'(k_C)$. At a steady-state equilibrium the amount of capital $K_H$ must grow at the rate of population growth. For the LHS of the above to remain finite,

$$\chi = (1 - \rho)\eta. \quad (17)$$

No capital gains accrue in housing if either $\rho = 1$ or $\eta = 0$; cf. Weiss (1978).

### Housing Is Produced with Capital and Labor

Let $(L_H)^\rho g(K_H/L_H)$ be the production function for new houses, where $g(\cdot)$ is assumed to be increasing and concave in its argument, $K_H$ and $L_H$ denote the total amount of capital and labor employed by the housing producing sector, respectively, and $\rho$ is the decreasing returns to scale parameter, $0 < \rho \leq 1$.\(^{14}\) We assume that capital used in the production of both goods, housing and non-housing, depreciates fully and that the housing stock does not depreciate.

The non-housing good can be used for both consumption and investment. For equilibrium, the total investment demand (that is, by both the housing producing sector and the non-housing producing sector) plus the demand for consumption must be equal to the total supply of the newly produced non-housing good. It is convenient to express the equilibrium conditions in intensive form. We denote the shares of labor and capital employed in the two sectors as $l_C \overset{\Delta}{=} L_C/L$, $l_H \overset{\Delta}{=} L_H/L$, and the sectoral factor intensities by $k_C \overset{\Delta}{=} K_C/L_C$, $k_H \overset{\Delta}{=} K_H/L_H$. The condition for equilibrium in the non-housing sector then becomes

$$k_C l_C + k_H l_H + C_D = l_C f(k_C). \quad (18)$$

The real rate of interest equalizes the rate of return to capital in both sectors:

---

\(^{14}\)In order for the marginal product of labor to be positive, it must be the case that $\rho$ exceeds the elasticity of $g(k_H)$ with respect to the capital-labor ratio, $k_H$. 

\[ P_H(0)e^{\xi t}L_H^{\rho-1}g'(k_H) = 1 + I = f'(k_C). \]  
(19)

The real wage \( W \) is determined so as to equilibrate the marginal product of labor in both sectors:

\[ P_H(0)e^{\xi t}L_H^{\rho-1}[\rho g(k_H) - k_Hg'(k_H)] = W = f(k_C) - k Cf'(k_C). \]  
(20)

Equilibrium in the labor market is written more conveniently in intensive form in terms of the labor shares \( l_H \) and \( l_C \):

\[ l_C + l_H = 1. \]  
(21)

Again, the rate of capital gains consistent with steady state equilibrium follows from the sectoral equilibrium condition. At the steady state, \( L_H \) and \( L_C \) are growing at a rate equal to \( \eta \), the growth rate of total labor supply, and so do \( K_H \) and \( K_C \). From (19) and (20) we have that, at the steady state, the rate of increase in the price of housing stock must satisfy (17).

We follow the standard Uzawa (1962–3) notation with two auxiliary variables: first, the economy-wide ratio of the wage rate to the rental rate of capital, \( \omega \),

\[ \omega \equiv \frac{W}{1+I}; \]  
(22)

and second, the economy-wide capital-labor ratio, \( k \),

\[ k \equiv l_Ck_C + (1-l_C)k_H. \]  
(23)

This notation enables us to express general equilibrium in our economy in terms of \((\omega, k, P_H(0); N)\) as the only unknowns.

We divide the condition that expresses the equilibrium allocation of labor across sectors, (20), by the condition that expresses the equilibrium allocation of capital across sectors, (19), to obtain

\[ \omega = \rho \frac{g(k_H)}{g'(k_H)} - k_H = f(k_C) - k_C. \]  
(24)

With \( g(\cdot) \) and \( f(\cdot) \) being concave, (24) defines the capital intensities in the two sectors as functions of the wage-capital rental ratios, \( k_C = k_C(\omega) \) and \( k_H = k_H(\omega) \). It follows that \( k_C(\omega) \) is increasing in \( \omega \). As for \( k_H(\omega) \), it is increasing (decreasing) in \( \omega \), if \( \rho(1 - \epsilon_k^C/\epsilon_k^H) - 1 \) is positive (negative).
Furthermore, it follows from (19) that $I$ can be written as a function of $\omega$, $I(\omega)$.

The market for new housing is at equilibrium, if $\hat{H}_D = (L_H)^\rho g(K/H/L_H)$. Or, after using (7), we have

$$\alpha_H \eta \rho \tilde{W}_0 \Psi[I(\omega), (1 - \rho)\eta, N] = P_H(0) \tilde{L}^{\rho - 1} I^\rho_H g(k_H),$$

(25)

where the auxiliary function $\Psi[\cdot]$, defined in (16), is now a function of $\omega$ and $N$ only.

We characterize general equilibrium of a steady state of this economy by noting first that the housing market equilibrium condition (25) yields an expression for the flow of housing profits per capita at a steady-state equilibrium:

$$\pi_0 = (1 - \rho)P_H(0) \tilde{L}^{\rho - 1} I^\rho_H g(k_H) = (1 - \rho)\eta \rho \alpha_H \tilde{W}_0 \Psi[I(\omega), (1 - \rho)\eta, N].$$

Income consists of labor income, income from rents and income from housing production, $f(k_C) - k_C f'(k_C) + [IP_H(t) H_D(t)/L(t)] + (1 - \rho) \eta \rho \alpha_H \tilde{W}_0 \Psi[I(\omega), (1 - \rho)\eta, N]$. Using the above expressions and writing $k_C$ and $I$ as functions of $\omega$, we solve to obtain an expression for $\tilde{W}_0$, at equilibrium, as a function of $(\omega; N)$ only:

$$\tilde{W}_0(\omega; N) \equiv \left(1 - [I + (1 - \rho)\rho \eta] \frac{1 - e^{-IT}}{I} \alpha_H \Psi[I, (1 - \rho)\eta, N]\right)^{-1} \times \left[f(k_C) - k_C f'(k_C) \frac{1 - e^{-IT}}{I}\right].$$

(26)

General equilibrium is fully described as a solution to the following set of simultaneous equations in terms of $\{k, \omega, P_H(0)\}$, given $N$, the number of moves over a person’s lifetime:

$$\alpha_c \Theta(\eta) \frac{1 - (I + (1 - \rho)\rho \eta)^{1 - e^{-IT}} / I \alpha_H \Psi[I, (1 - \rho)\eta, N]}{1 - (I + (1 - \rho)\rho \eta)^{1 - e^{-IT}} / I \alpha_H \Psi[I, (1 - \rho)\eta, N]}$$

$$= \left[f(k_C(\omega)) - k_C(\omega)f'(k_C(\omega))\right]^{-1} \left(\frac{k - k_H(\omega)}{k_C(\omega) - k_H(\omega)} f(k_C(\omega)) - k\right);$$

(27)

$$P_H(0) \tilde{L}^{\rho - 1} \left(\frac{k_C(\omega) - k}{k_C(\omega) - k_H(\omega)}\right)^{\rho - 1} g'(k_H(\omega)) = f'(k_C(\omega));$$

(28)
Here \( (27) \) is the counterpart of \( (11) \) for a growing economy, the equilibrium condition for the economy-wide capital-labor ratio \( k \), \( (28) \) expresses inter-sectoral equilibrium in terms of the economy-wide factor-price ratio \( \omega \), and \( (29) \) expresses housing market equilibrium.

Equations \( (27) \) and \( (29) \) assume that \( N \) is given. If moves are unrestricted, then \( N = \mathcal{N} (1 - (1 - \rho)\eta, B / \alpha_H) \). In that case, equations \( (27) - (29) \) yield the Walrasian equilibrium vector \( (k, \omega, P_H(0)) \). This is the growing economy counterpart of the unrestricted case in Section III. From the solution for \( (k, \omega, P_H(0)) \) of the system of simultaneous equations \( (27) - (29) \), capital intensities in the two sectors follow readily from their definitions, as do the allocations of labor to the sectors.

From \( (29) \) and \( (27) \) we can solve for \( \alpha_H \Psi() \); by substituting back into \( (29) \) we eliminate the impact of residential mobility. By eliminating \( P_H(0) \) between \( (28) \) and \( (29) \) we obtain the two-sector counterpart of the factor-price frontier, a relationship between the economy-wide capital-labor ratio \( k \) and the factor-price ratio \( \omega \). This relationship encapsulates the feasible tradeoffs between capital intensity and the factor-price ratio which are made possible in spite of the presence of residential mobility.\(^{15}\)

The general equilibrium system \( (27) - (29) \) accomplishes two things: first, it generalizes the Cass-Yaari model to a two-sector model. The general equilibrium system with growth now unfortunately lacks a recursive structure (which it had in the case of constant population), even though it remains similar to the Cass-Yaari model. Second, and quite importantly, it does allow us to analyze the impact of restrictions in the housing market (such as rent controls) on the entire economy.

### Restrictions in the Housing Market

We now consider the case where residential moves are restricted and \( P_H(0) \) is arbitrarily set at less than its Walrasian level. First we note that unless

\(^{15}\)The solution must satisfy the condition \( I > \chi = (1 - \rho)\eta \), under which our formulas have been derived. A sufficient condition for this is that the steady-state equilibrium in this model is dynamically efficient in the sense of Diamond (1965): \( I - \eta > 0 \).
\( \chi = (1 - \rho)\eta \), a steady-state equilibrium will not exist. Under the assumption that \( P_H(t) \) grows at the rate \( \chi = (1 - \rho)\eta \), a steady-state equilibrium may exist. With \( P_H(0) \) being given, the number of moves \( N \) now replaces \( P_H(0) \) as a variable to be determined at the steady-state equilibrium.

As in the one-sector model, the nature of equilibrium in the housing sector affects the equilibrium rate of interest. Determination of the rate of interest is not separable from determination of the other unknown variables at equilibrium. When \( P_H(0) \) is prespecified, the equilibrium rate of interest depends on it, as does the restricted (non-Walrasian) equilibrium value of residential moves.

Evaluation of the impact of restrictions of the rent-control type is amenable to analysis similar to that proposed above for the case of no population growth. Housing sector reform in the case of the our two-sector model with a growing population will directly affect welfare at the steady state through the number of moves, given the economy-wide capital-labor and factor-price ratios. It will have an indirect \textit{general equilibrium} effect through the change in the equilibrium capital intensities and factor prices in the two sectors of the economy.

It is an open question whether housing sector reform through liberalization of the operation of housing markets is likely to have beneficial consequences for long-run economic growth. As is usually the case in overlapping-generations models, the direction of the effect depends on whether the competitive equilibrium is dynamically efficient in the model that constitutes the fundamental building block of this paper. However, as we mentioned above in the case of constant population, in this model efficiency would have to considered afresh.

\textit{Technological Change}

It is straightforward to allow for exogenous technological change and to consider the existence of balanced growth paths instead of steady states. Capital gains would continue to accrue to housing in the presence of exogenous, labor-augmenting technological change according to \( A(t) = A(0)e^{at} \), that affects labor in both sectors. In that case, \( \chi = (1 - \rho)(\eta + a) \).

Perhaps more interesting is the case of exogenous, labor-augmenting technological change, at a rate \( a \), in the production of the non-housing good only, arguably a reasonable assumption. Innovations in building technologies lag behind those in manufacturing and services. If we assume a homogeneous production function for new housing \( G(K_H) = K_H^{\gamma \rho} L_H^{(1-\gamma)\rho} \), \( 0 < \gamma < 1 \), the wage rate per efficiency unit of labor will be constant, if \( \chi = (1 - \rho)\eta + a(1 - \gamma \rho) \).

V. Conclusions

In an era of market liberalization, housing restrictions are typically among the last to go. The novel question addressed by this paper is what happens to the long-run growth prospects in such an economy when the housing market is subject to restrictions. Whether or not population is growing, housing market reform does affect the equilibrium allocation associated with long-run growth. If we restrict our setting to parameter values which ensure a unique steady-state equilibrium, then the lifting of restrictions lowers the steady-state capital-labor ratio. Whether welfare increases, however, depends on whether the steady-state equilibrium is inefficient.

The dynamic model of this paper can be used to help us understand how the behavior of market economies is affected by institutional factors which underlie the striking differences in rates of residential mobility between countries and regions. There are features which affect the cost of moving and hence households’ propensity to move, like the tax treatment of renters, landlords and owner-occupiers, and of capital gains from housing and other assets, links between local and national job and housing markets, the role of real-estate brokers, the transaction costs of residential moves, and the availability of housing finance. The indirect impact of such features on economic growth has received undeservedly little attention. Examination of such factors within this model and its further development outside steady states, cf. Ortalo-Magne and Rady (1997), would help explain the richness of housing market dynamics across countries; cf. Englund and Ioannides (1997).

References


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